Modeling Electricity Price Elasticity of Demand Using Heterogeneous Panel Data

Presentation of Draft Working Paper

Developed by the

Kentucky Energy and Environment Cabinet

Department for Energy Development and Independence

In Volunteer Partnership with the

University of Kentucky

Department of Statistics

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Executive Summary

In an effort to develop and validate electricity demand forecasting models, including electricity price elasticity of demand (PED) coefficients, this study modeled the responsiveness of electricity consumption across the United States to changes in the real price of electricity, weather, population, and income from 1970 to 2010 and then evaluated the performance of those models from 1970 to 2011.

This study quantified the degree to which consumer sensitivity to changes in electricity prices varies by both place and time after controlling for the numerous other exogenous factors driving sector-specific electricity demand in each state. The significant spatial and temporal heterogeneity of PED coefficients identified in this study suggests that imposing either homogenous or stationary PED coefficients may lead to biased electricity demand forecasts.

Four unique statistical methodologies were used to develop custom electricity demand models for each economic sector in each of the 48 contiguous United States; Ordinary Least Squares (OLS), Generalized Least Squares (GLS) with Fixed Effects (FE), Mixed Fixed and Random Coefficient Model (RCM or MFR) with Fixed Effects, and Time-Varying Coefficient Model (VCM). For each method, the limitations, assumptions, and ability to predict historical demand are discussed.

While the dependent variable here is electricity demand, these same quantitative methods may be used to forecast other electricity-price dependent factors of interest to the Commonwealth, including but not limited to, economic growth and employment.

Overview

Introduction

Review of Price Elasticity of Demand 4

Description of Data Analyzed 7

Summary of Estimated Price Elasticity of Demand Coefficients for Kentucky 9

Discussion of Model Evaluation

Ordinary Least Squares Model 12

Generalized Least Squares Model with Fixed Effects 18

Random Coefficient Model with Fixed Effects 28

Varying Coefficient Model with Fixed Effects 41

Comparison of Model Results 49

An interactive spreadsheet version of the residential demand model is available for download at:
http://energy.ky.gov/Programs/Data%20Analysis%20%20Electricity%20Model/Residential_Electricity_Demand_Model.zip

Review of Price Elasticity of Demand

Review of Price Elasticity of Demand

$$\textbf{Electricity Price Elasticity of Demand} = \frac{\textbf{\% Change in Electricity Demand}}{\textbf{\% Change in Electricity Price}}$$

Formally, electricity Price Elasticity of Demand (PED) coefficients are the percentage change in the quantity of electricity demanded given a percentage change in the price of electricity, ceteris paribas.

Informally, PED's can be understood as estimates of consumer sensitivity and responsiveness to changes in electricity prices, holding all other factors constant.

PED's are bidirectional, such that a decrease in electricity prices would increase demand.

Review of Price Elasticity of Demand

Price Elasticity of Demand Spectrum

PED's are generally understood to be negative as demand should logically have an inverse relationship with price, as implied by the Law of Demand.

Electricity PED's typically fall in the range of relative inelasticity, with short-term elasticity estimates substantially less-responsive than long-term elasticity, as the ability of consumers to implement efficiency and adopt alternative energy sources increases with the duration of price increases.

While elasticities are usually divided into short-term and long-term, for simplicity, this study focuses specifically on long-term elasticity.

Data Analyzed

Data Analyzed

Unit of Observation
48 Contiguous United States (Alaska, Hawaii, and District of Columbia Excluded)

Timeframe
Annual Data from 1970 to 2010
Monthly data is also being modeled but, for simplicity, is not discussed in this presentation.

Dependent Variables
Electricity Consumption by Consumer Class (Commercial, Industrial, & Residential)

Independent Variables of Interest
Real Electricity Price by Consumer Class (Commercial, Industrial, & Residential)

Control Variables Included
Weather (Cooling Degree Days & Heating Degree Days)
Natural Gas Price by Sector as a Fuel Substitute
Socioeconomic Factors: Population, Income
Time

All included variables were converted to their natural logarithms such that the resulting coefficients may be simply interpreted as elasticities.

All prices converted to real 2010 $US, i.e. inflation adjusted using the Bureau of Labor Statistic's Consumer Price Index.

Estimated PED Coefficients

Estimated PED Coefficients

Kentucky Long-Term PED Coefficient Estimates, 2011				
	Homogeneous Models		Heterogeneous Models	
Sector	**OLS**	**VCM**	**FE**	**RCM**
Commercial	-0.5363	-0.5028	-0.2052	-0.3087
Industrial	-0.9434	-1.2241	-0.3373	-0.4126
Residential	-0.8609	-0.7637	-0.3097	-0.5319

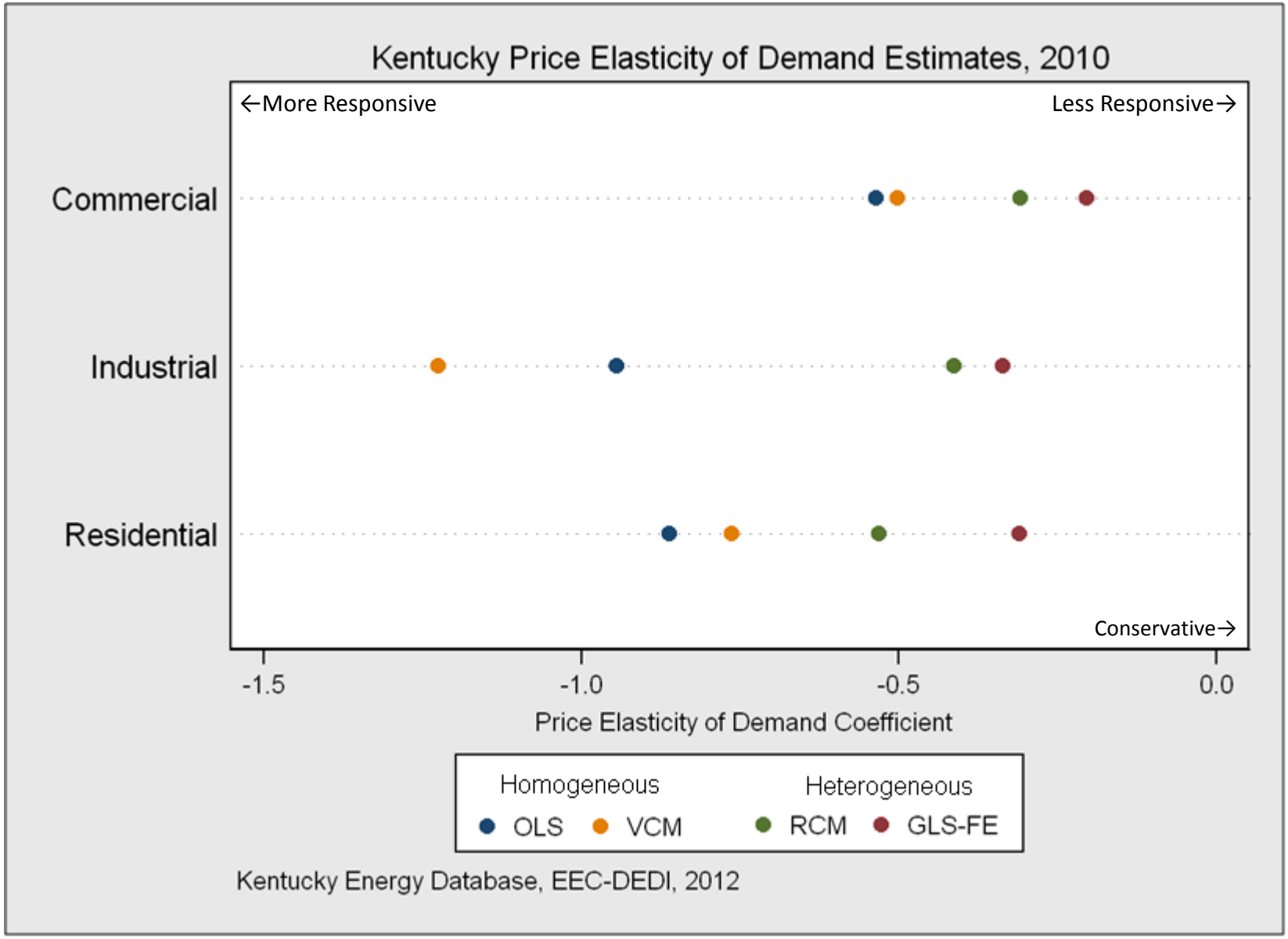
These electricity price elasticity coefficients suggest that, given a 10% increase in the real price of electricity in Kentucky, and holding all other included factors constant:

- Commercial consumers would reduce electricity consumption by between 2.1% and 5.4%.
- Industrial consumers would reduce electricity consumption by between 3.4% and 12.2%.
- Residential consumers would reduce electricity consumption by between 3.1% and 8.6%.

Furthermore, this study demonstrates that allowing for spatial (i.e. between cross-section) heterogeneity in either demand (i.e. the Fixed Effects Model) or both demand and price elasticity (i.e. the Mixed Fixed and Random Coefficients Model) leads to more conservative, (i.e. less responsive) elasticity coefficients, whereas imposing homogeneity may bias demand forecast results and exaggerate demand responsiveness to changes in price.

All models discussed here were developed using data fro the 48 contiguous United States. PED coefficients for all 48 states are provided on p. 34 with complete models on p. 50.

Estimated PED Coefficients



OLS Model

OLS Model

$$Y_{it} = \beta_0 + \sum_{j=1}^{k-1} \beta_j X_{jit} + \varepsilon_{it}$$

$$i = 1, \ldots, 48 \ (state)$$

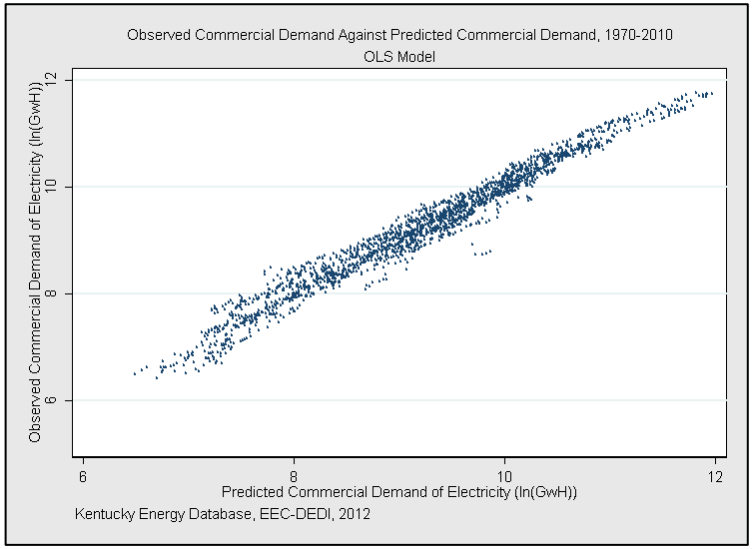
$$t = 1970, \ldots, 2010 \ (year)$$

$$k = 8$$

The Ordinary Least Squares (OLS) regression model minimizes the sum of squared differences between observed factors and the response predicted by linear estimation. This classic model was applied to data for all 48 contiguous states, which produced one model common to all states. OLS does not recognize the space and time dimensionality of panel data. The OLS model can be given by the above formula where *i* and *t* index states and years, such that y_{it} is the dependent variable of interest, electricity consumption by economic sector, in state *i* in year *t*, β_0 is the constant *y* intercept across all states, *X* is a *k* by 1 vector of explanatory variables, $\beta_j X_{jit}$ is the product of the observation for each independent variable *j* through *k* for state *i* in year *t* and the coefficient of *X*, *k* is the total number of included independent variables, and ε_{it} are the residuals, and where $\varepsilon_{it} \sim N(0, \sigma^2)$, or are approximately normally distributed with a mean of zero.

OLS Model – Observed vs. Predicted

Commercial



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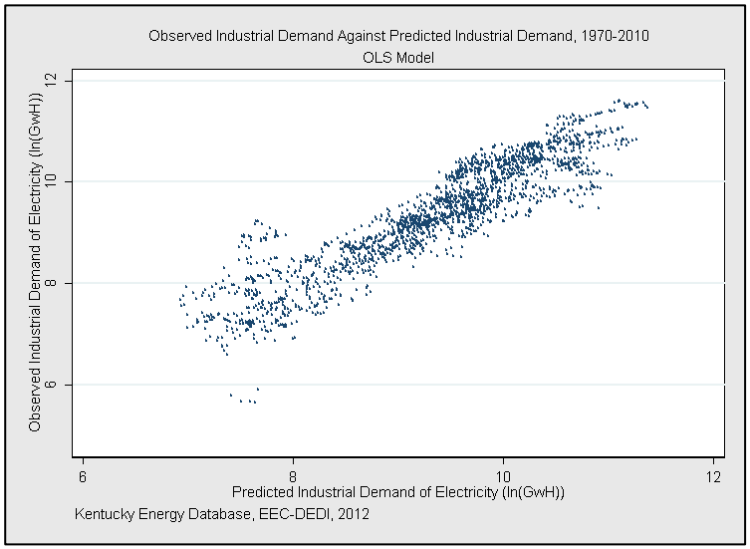
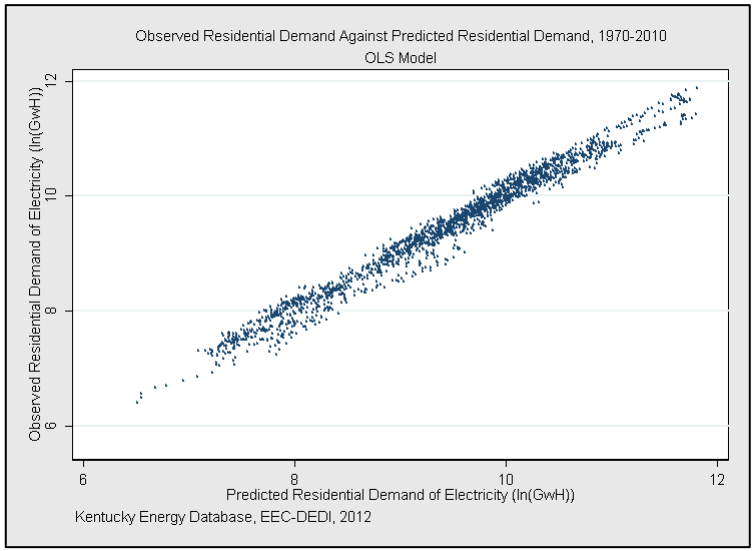


Residential



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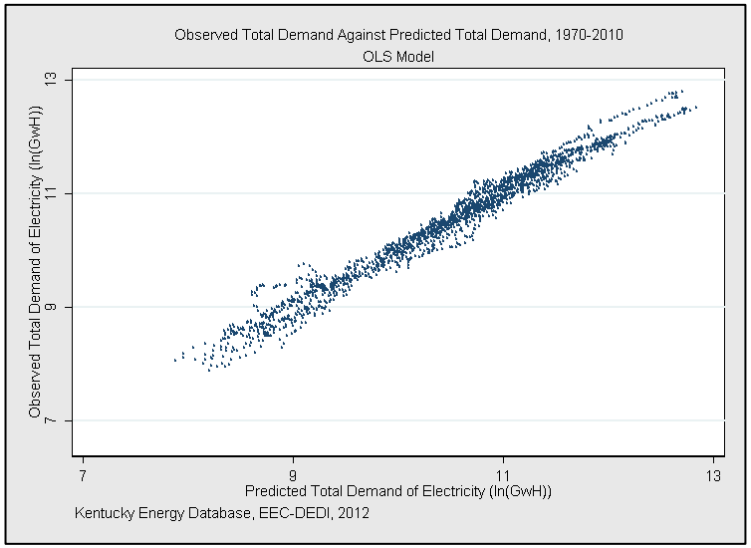
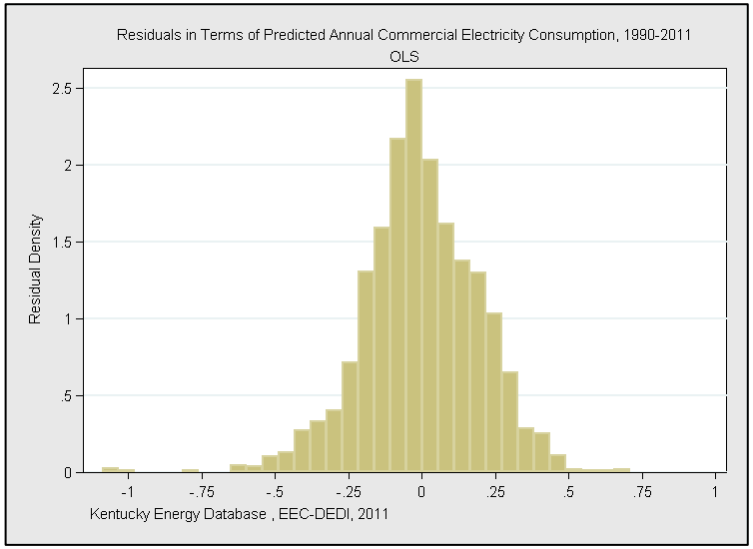


OLS Model – Residual Histogram

Commercial



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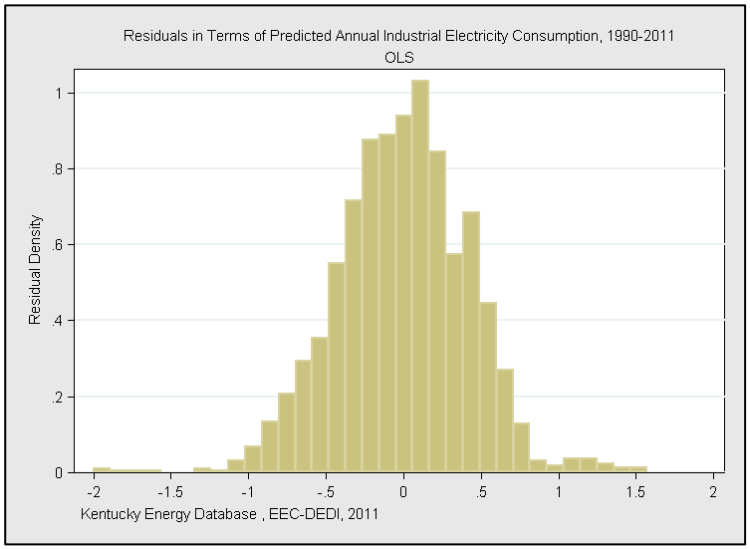

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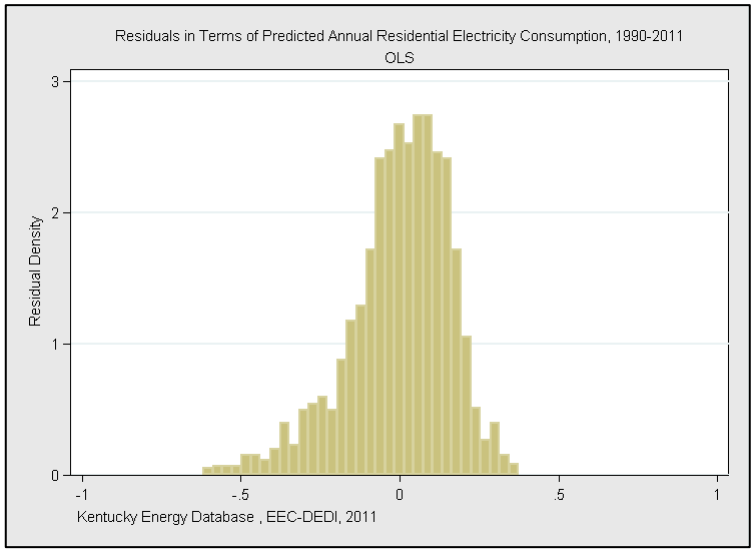

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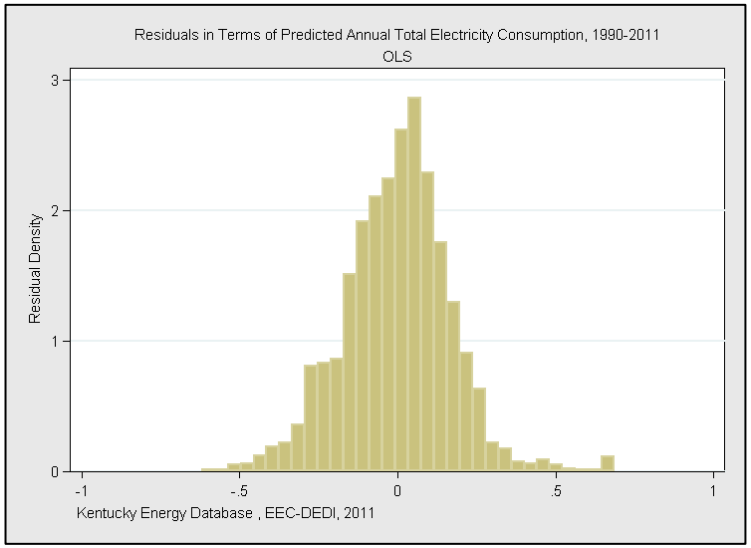
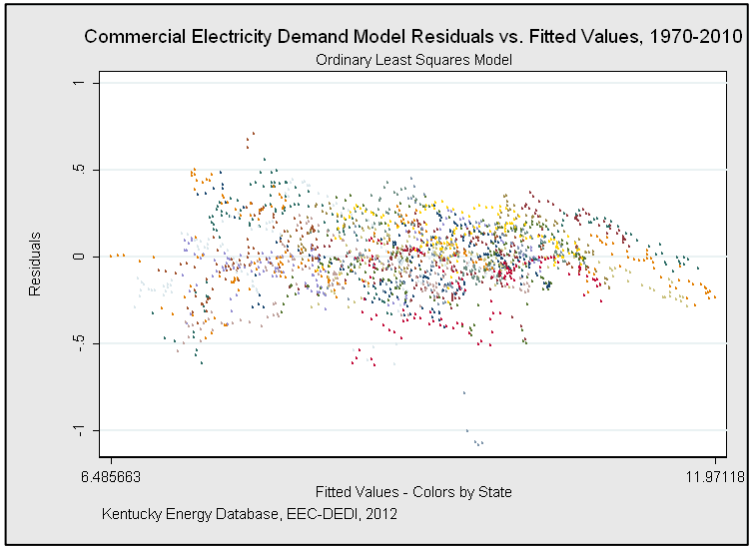


OLS Model – Residual Scatter Plots

Commercial



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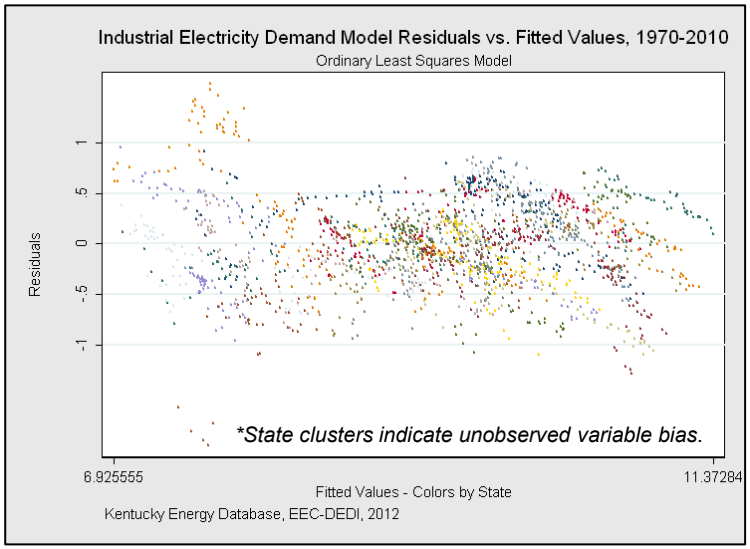

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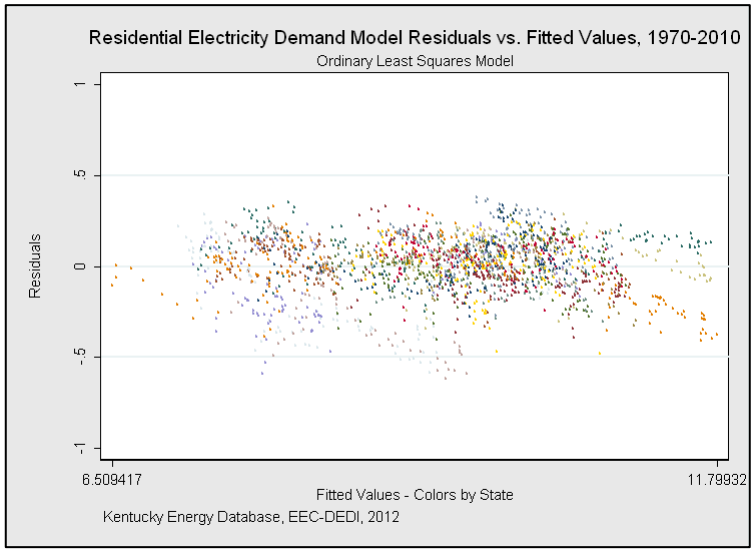

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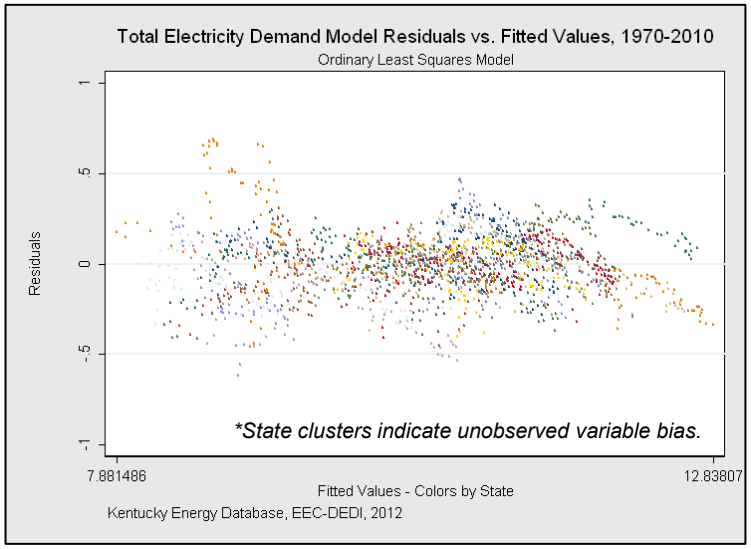
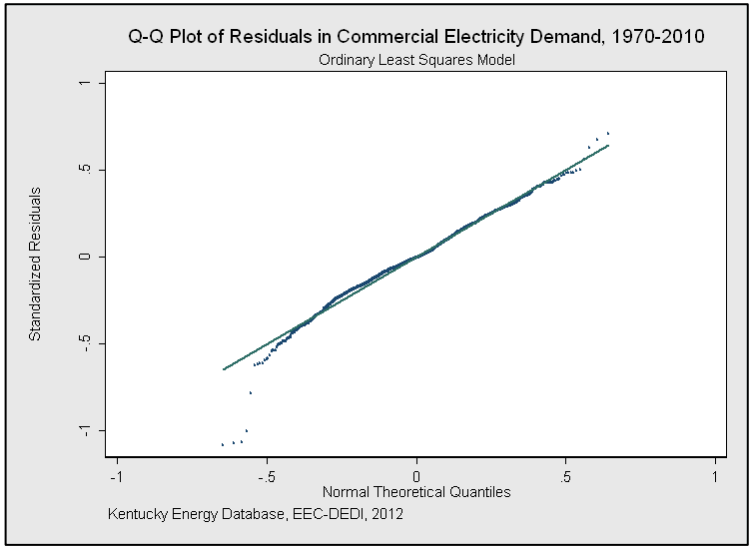


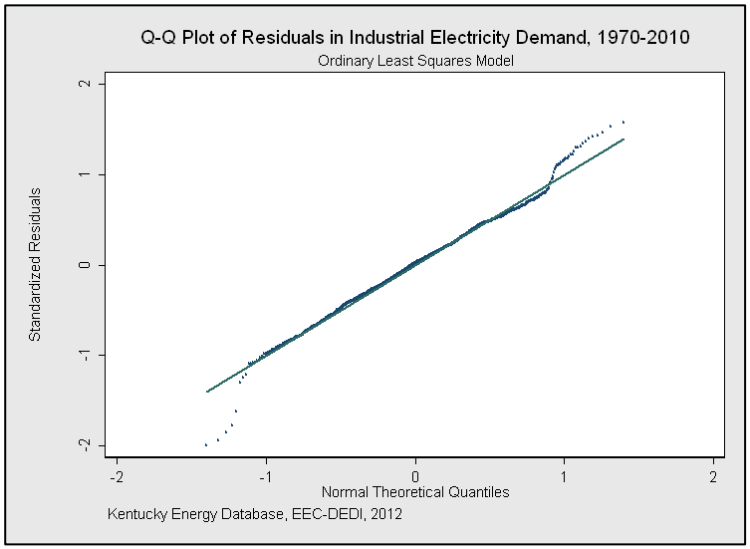
**Heteroscedasticity Identified*

OLS Model – Q-Q Plots

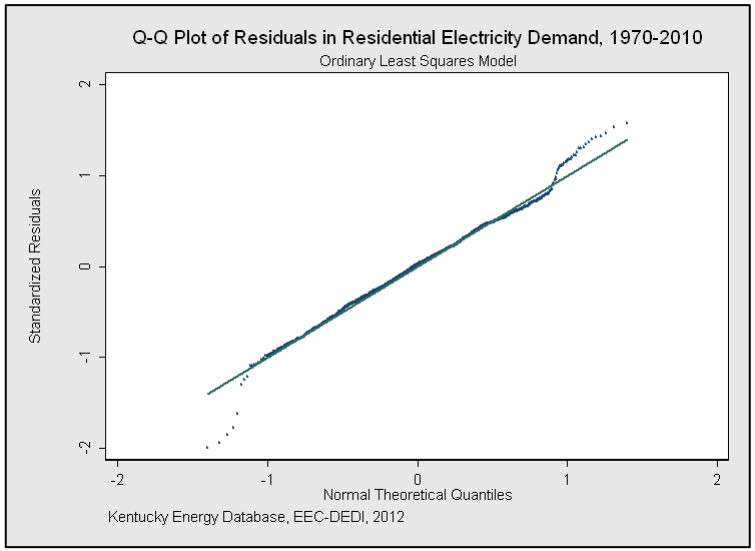
Commercial



Industrial



Residential



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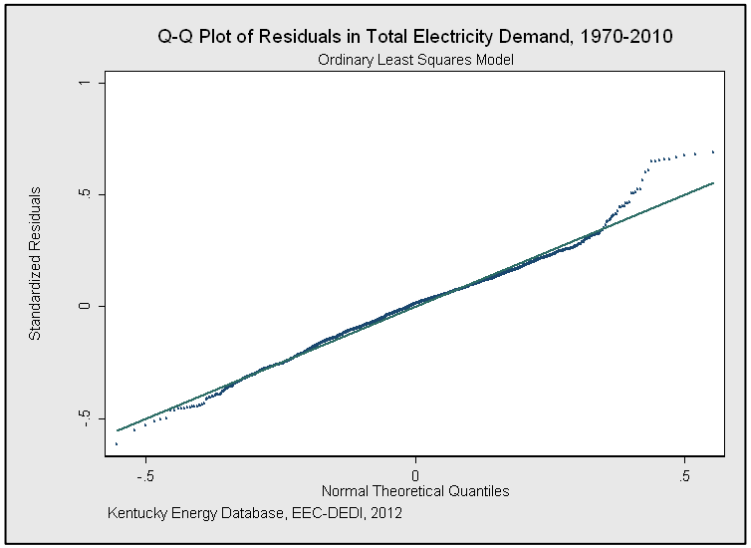


Fixed Effects Model

Fixed Effects Model

$$Y_{it} = \beta_0 + \sum_{j=1}^{k-1} \beta_j X_{jit} + \alpha_i + \varepsilon_{it}$$

$$i = 1, \ldots, 48 \ (state)$$
$$t = 1970, \ldots, 2010 \ (year)$$
$$k = 8$$

Where i and t index states and years, such that y_{it} is the dependent variable of interest, electricity consumption by economic sector, in state i in year t, β_0 is the constant y intercept across all states, X is a k x 1 vector of explanatory variables, $\beta_j X_{jit}$ is the product of the observation for each independent variable j through k for state i in year t and the coefficient of X, k is the total number of included independent variables, α_i is the time-invariant fixed effect for state i, and ε_{it} are the residuals, and where $\varepsilon_{it} \sim N(0, \sigma^2)$, or are approximately normally distributed with a mean of zero.

Fixed Effects Model

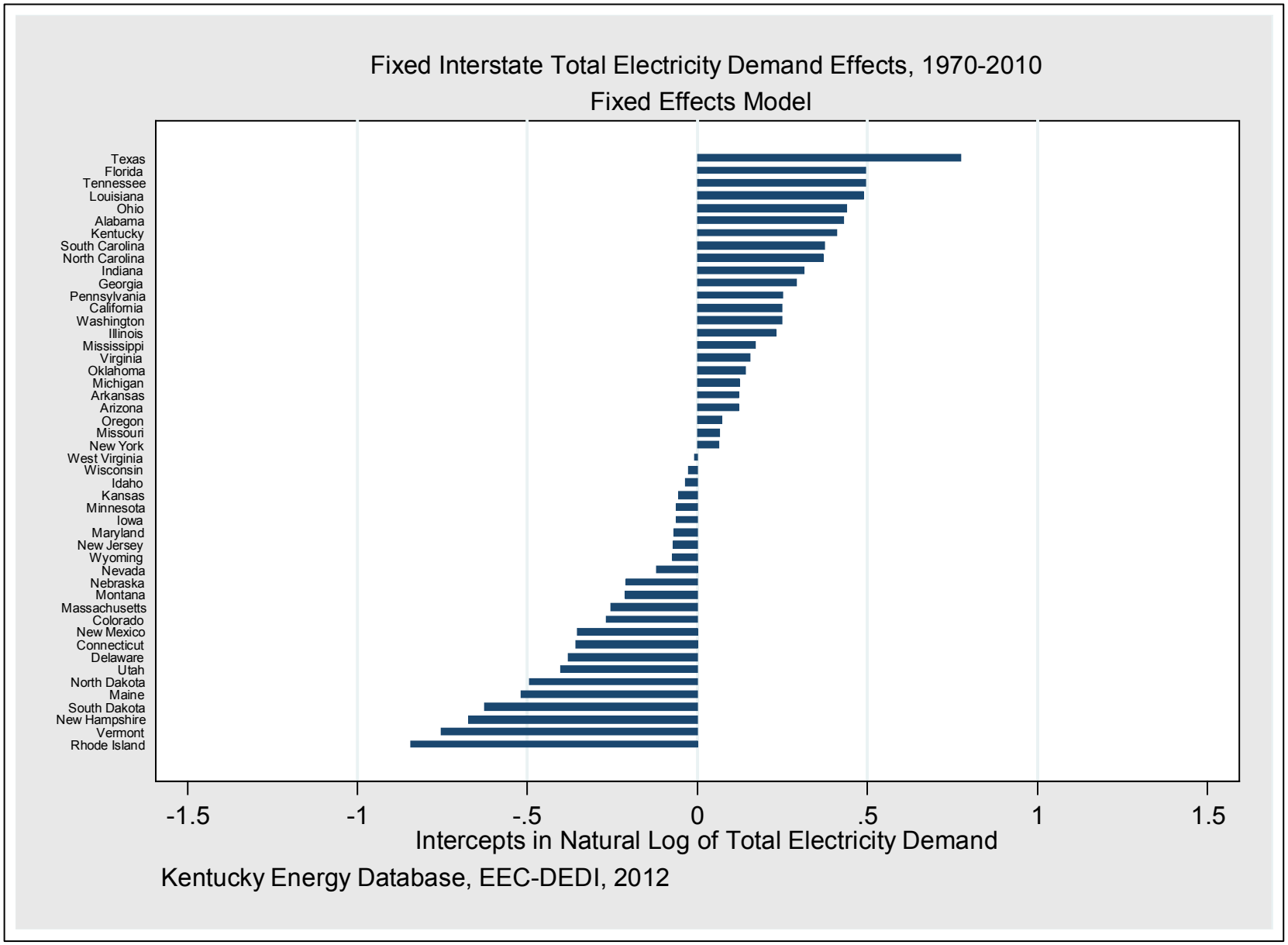
This model uses a statistical analysis technique called multiple regression of panel data with fixed effects, which builds upon Ordinary Least Squares (OLS) regression by isolating the time-independent constant difference in electricity demand (y_{it}) between cross-sectional units (states or *i*) that are correlated with the explanatory variables (X_{jit}).

A fixed effects model specifically assumes the existence of unobserved time-invariant heterogeneity in electricity demand between units, often referred to as unobserved variable bias, which in addition to the included independent variables, are affecting the dependent variable (demand). The fixed effects model then attempts to control for these missing or unobserved between unit (interstate) factors, the fixed effects (α_i), to isolate the specific net effect of the independent variables (β_j)on all cross-sections (nationally).

Therefore, as with OLS, the resulting PED coefficient is uniform across all states and times. The resulting forecast may; thus, be biased since both spatial and temporal homogeneity has been imposed on the coefficients.

However, since a custom fixed effect for demand is calculated for each state, demand forecasts are significantly improved versus OLS by adjusting electricity consumption to best fit the model to historical data. The PED coefficients from a model with fixed effects tend to be smaller (i.e. less responsive) as some of the estimated significance of electricity prices moves toward a fixed difference in electricity demand between states.

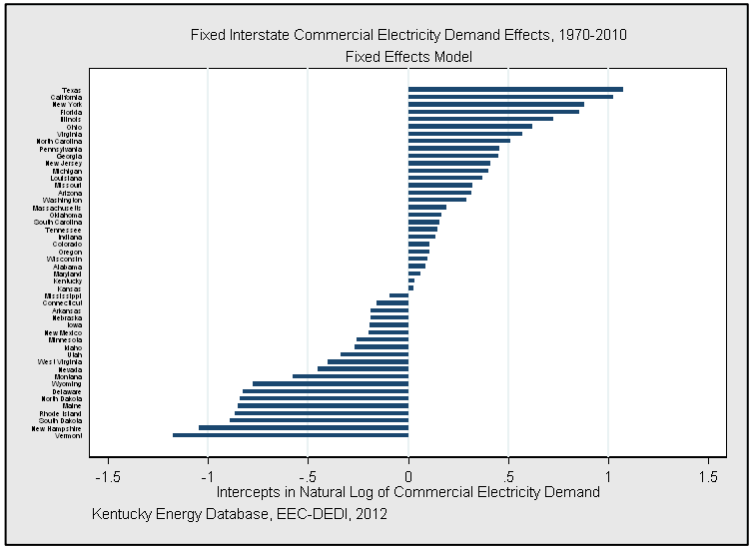
Fixed Effects for Demand (α_i)



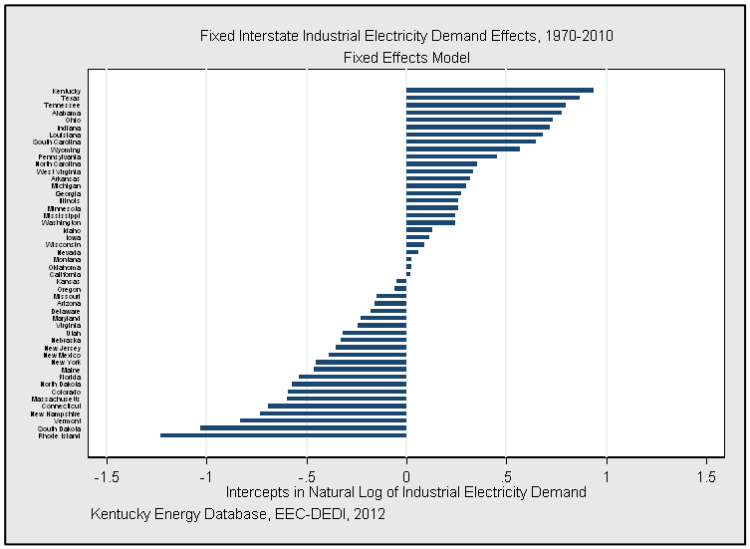
States where the natural log of electricity demand is higher or lower than expected even after controlling for all of the independent variables.

Fixed Effects for Demand (α_i)

Commercial



Industrial



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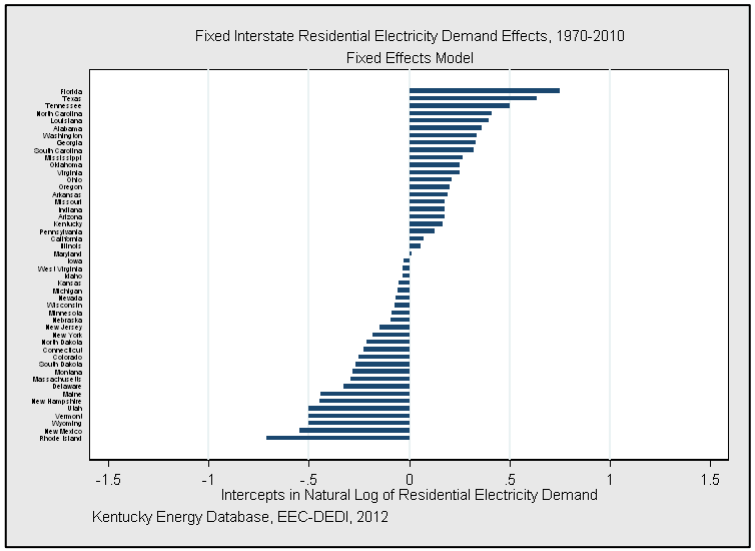

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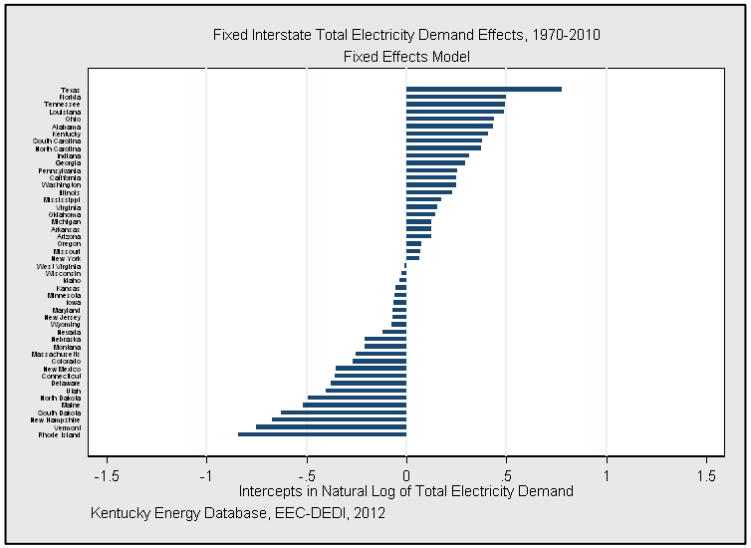
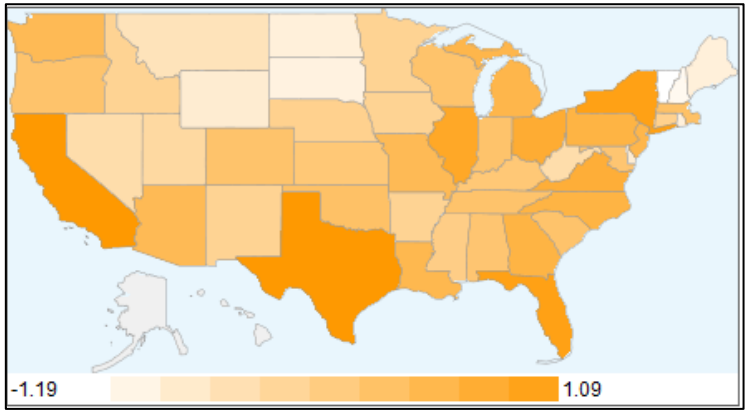


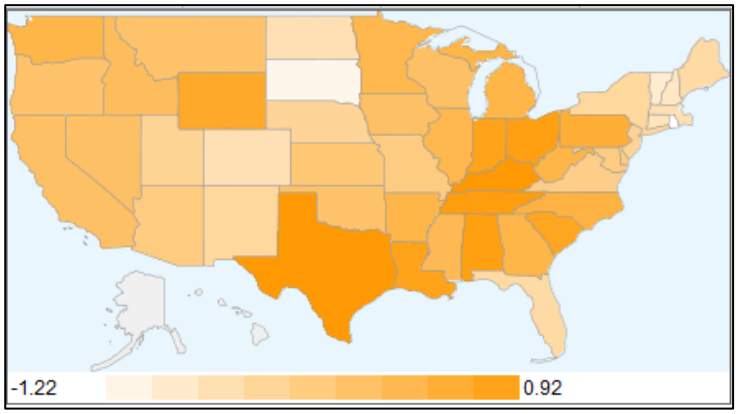
States where the natural log of electricity demand is higher or lower than expected even after controlling for all of the independent variables.

Fixed Effects for Demand (α_i)

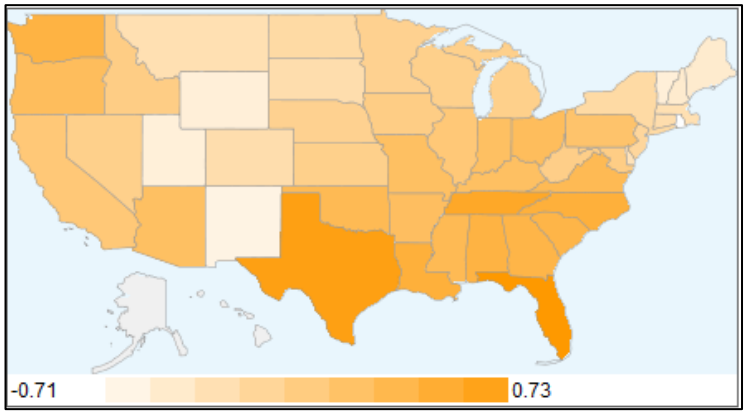
Commercial



Industrial



Residential



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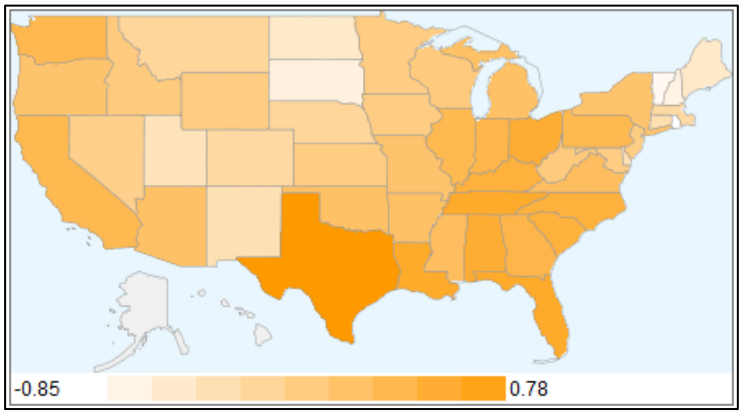
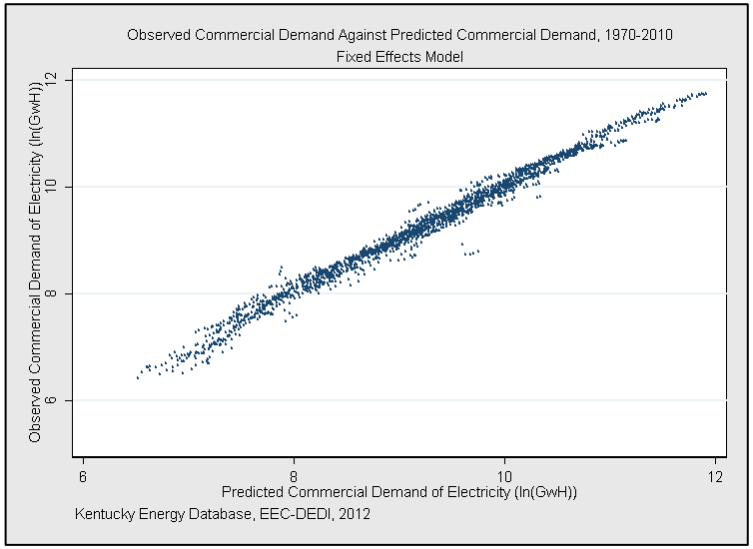


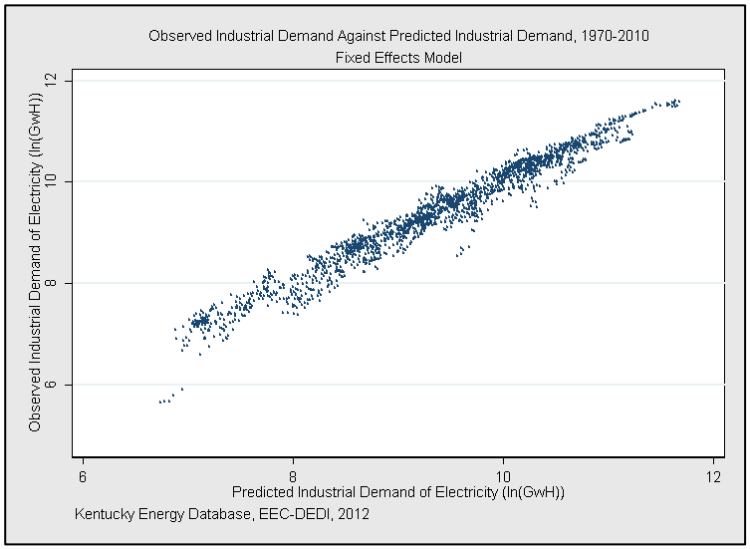
States where the natural log of electricity demand is higher or lower than expected even after controlling for all of the independent variables.

FE Model – Observed vs. Predicted

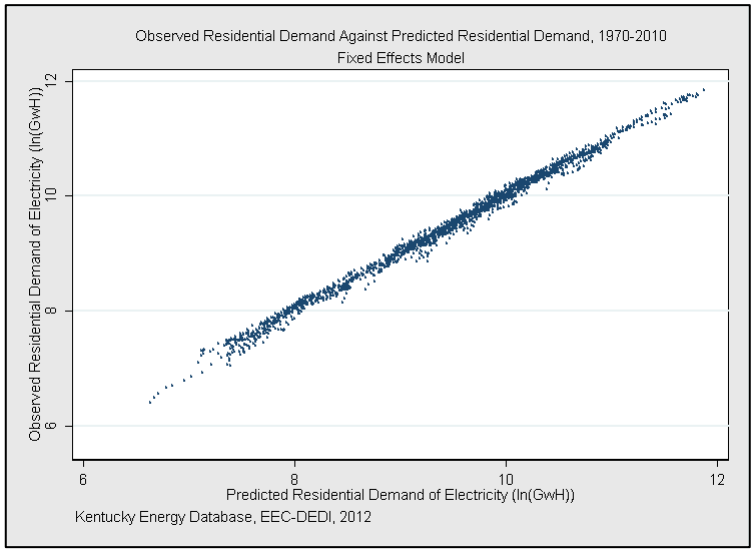
Commercial



Industrial



Residential



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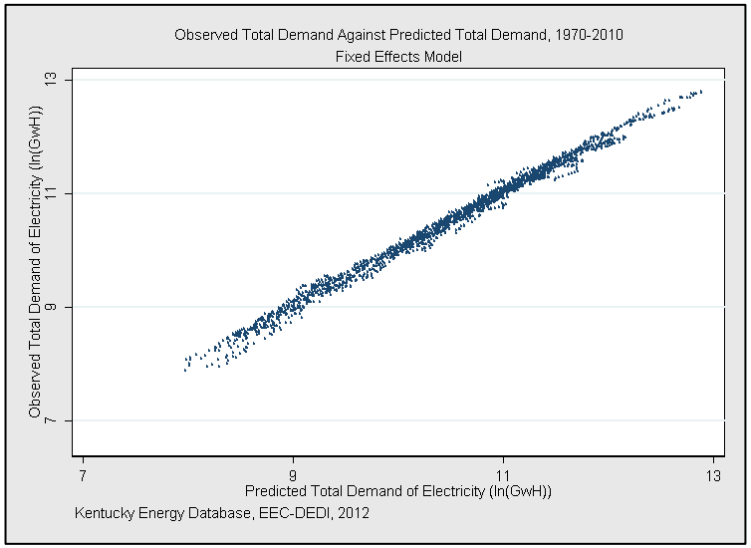
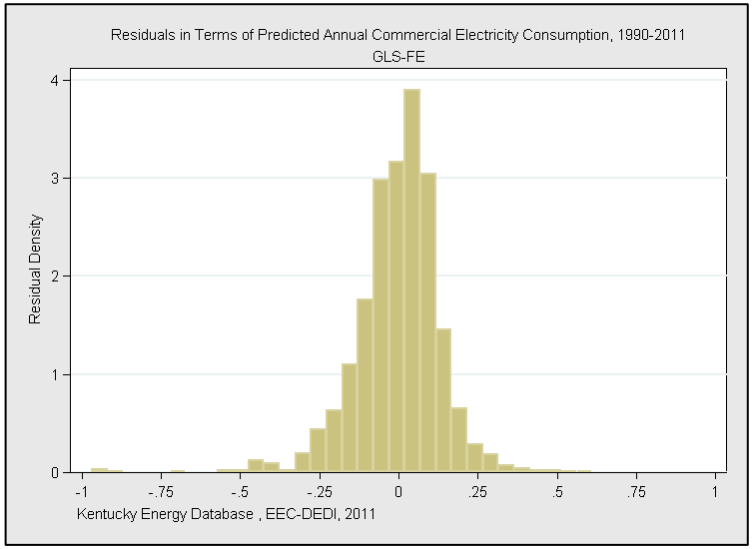


FE Model – Residual Histograms

Commercial



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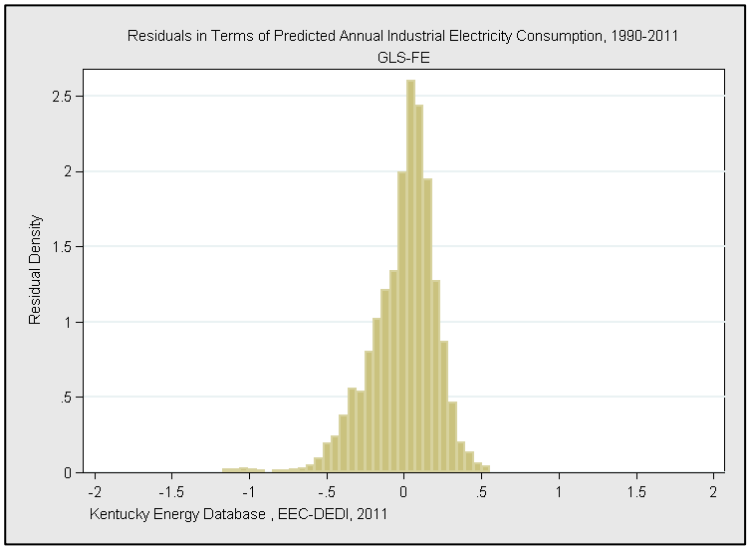
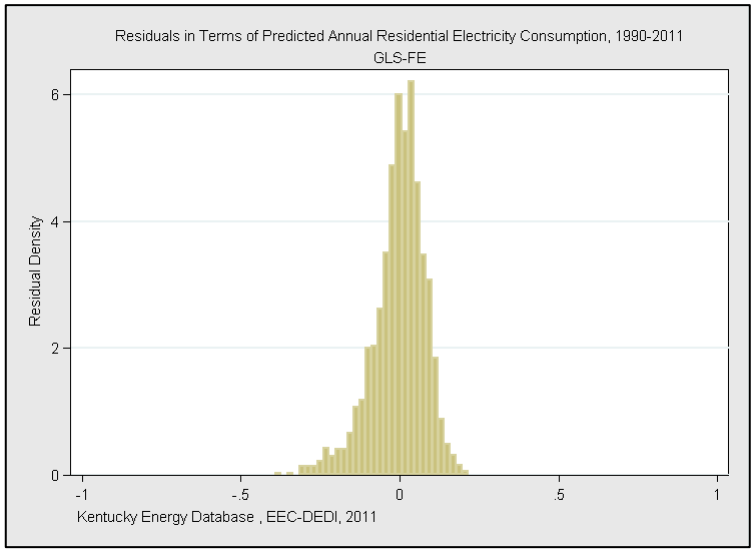


Residential



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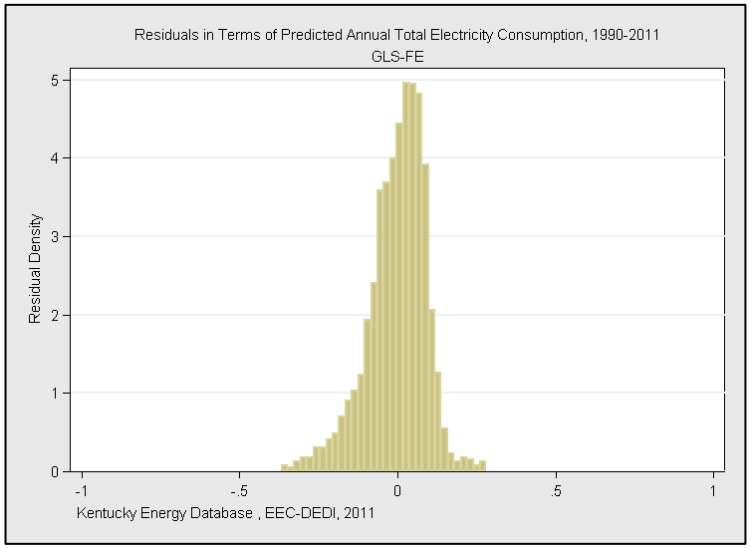
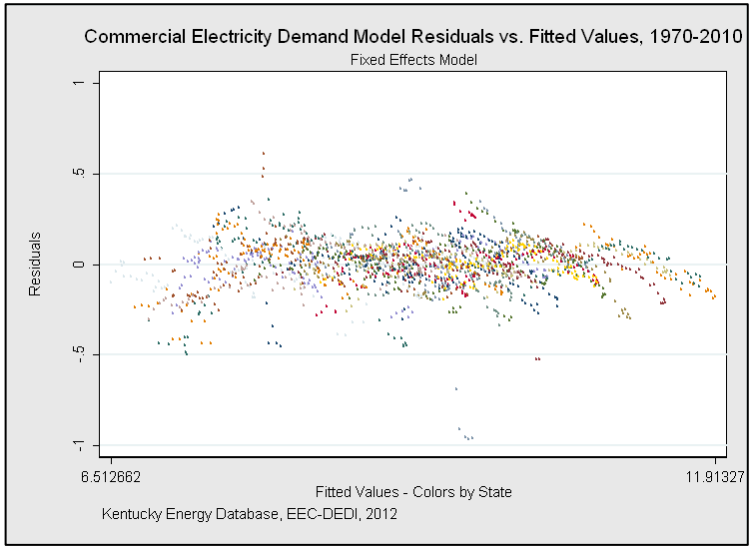
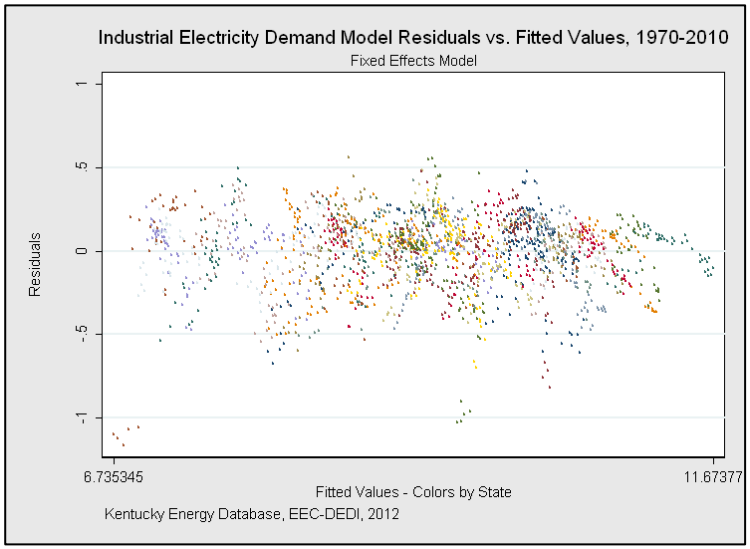


FE Model – Residual Scatter Plots

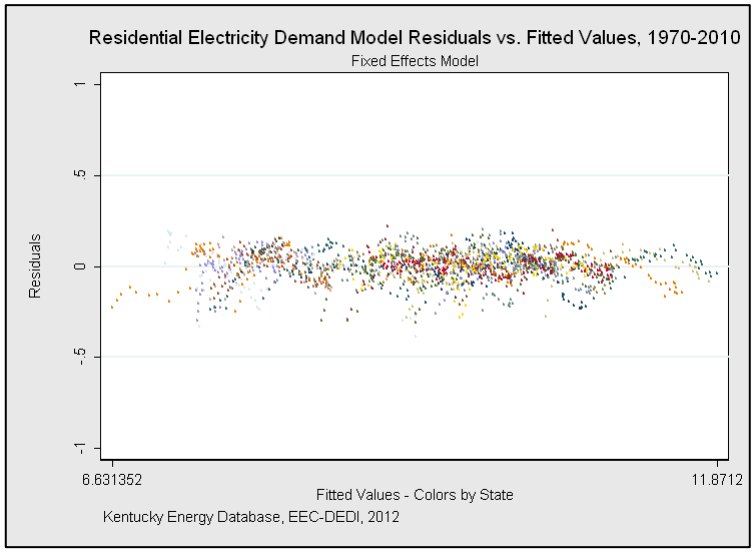
Commercial



Industrial



Residential



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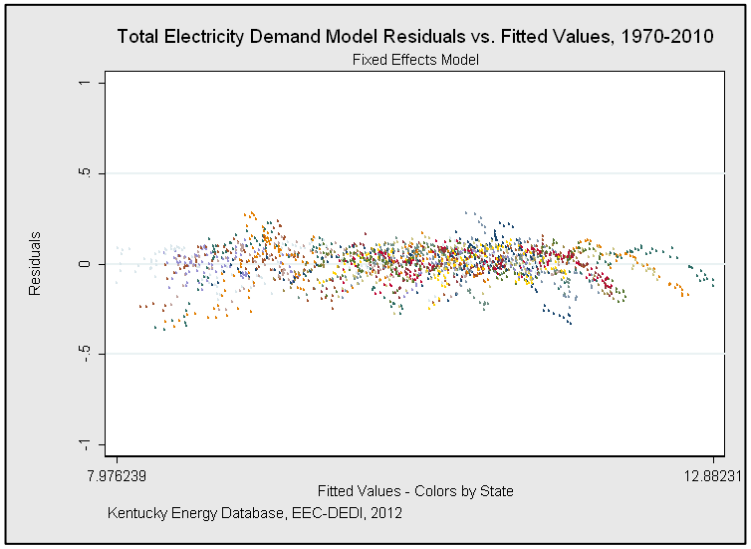
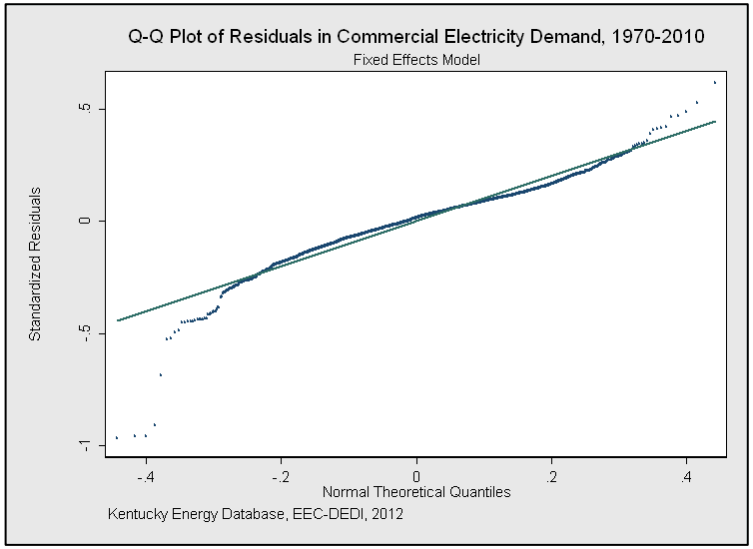


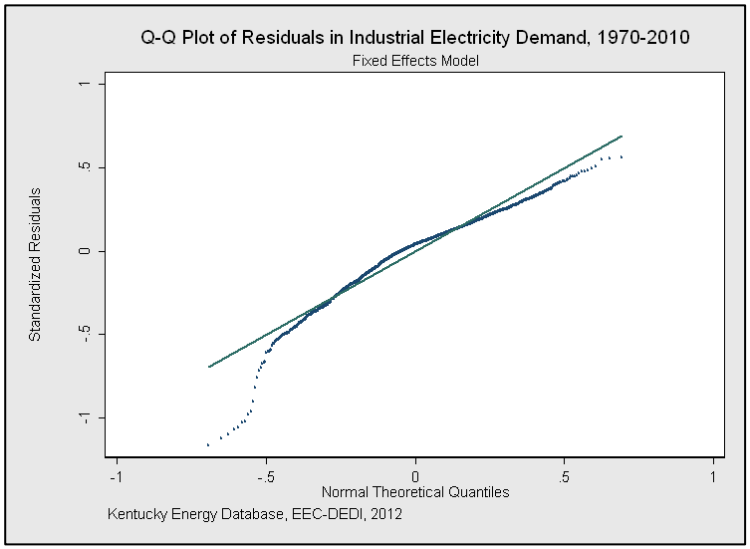
**Homoscedastic & Unbiased*

FE Model – Q-Q Plots

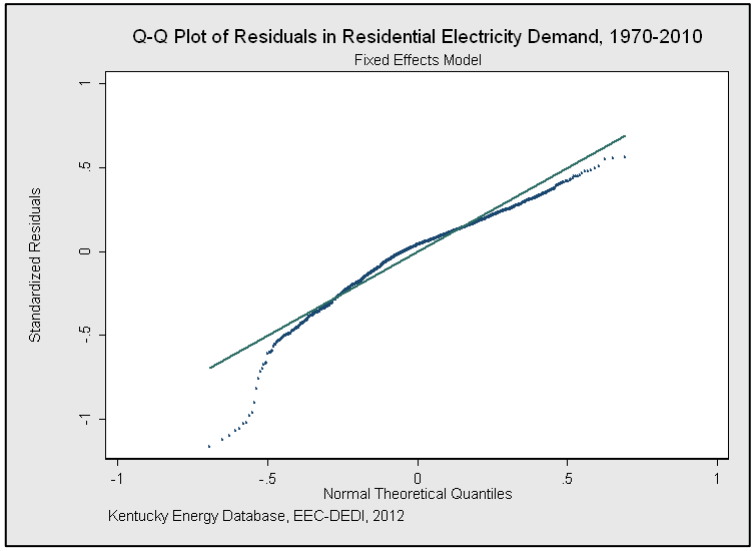
Commercial



Industrial



Residential



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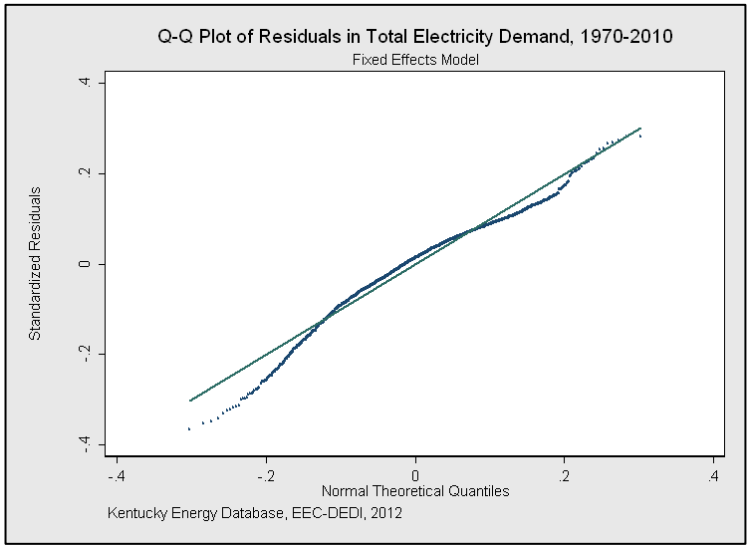


Random Coefficient Model

RCM Model

$$Y_{it} = \beta_0 + \sum_{j=1}^{k-1} \beta_j X_{jit} + \beta_{1i} X_{1it} + \alpha_i + \varepsilon_{it}$$

$$i = 1, \ldots, 48 \ (state)$$
$$t = 1970, \ldots, 2010 \ (year)$$
$$k = 8$$
$$X_{1it} = \ Price$$

Where *i* and *t* index states and years, such that y_{it} is the dependent variable of interest, electricity consumption by economic sector, in state *i* in year *t*, β_0 is the constant *y* intercept across all states, *X* is a *k* x 1 vector of explanatory variables, $\beta_j X_{jit}$ is the product of the observation for each independent variable *j* through *k* for state *i* in year *t* and the coefficient of *X*, $\beta_{1i} X_{1it}$ is the product of time invariant PED and electricity price for different states, *k* is the total number of included independent variables, α_i is the time-invariant fixed effect for state *I*, and ε_{it} are the residuals, and where $\varepsilon_{it} \sim N(0, \sigma^2)$, or are approximately normally distributed with a mean of zero. (Hsiao, Cheng et al. 1989)

RCM Model

The Mixed Fixed and Random Coefficient Model, referred to here as an RCM, in addition to estimating a time-invariant fixed effect (α_i) for electricity demand for each state, also estimates the degree of homogeneity and heterogeneity of coefficients between cross sectional units. Such that, there is a certain portion of price sensitivity common to all states (β_1) and another that is allowed to vary from state to state (β_{1i}), while the remainder of the coefficients for the control variable in the model ($\beta_2 \ldots \beta_k$) remain homogeneous. This model is, thus, a mixed model with both fixed and random coefficients as well as fixed effects.

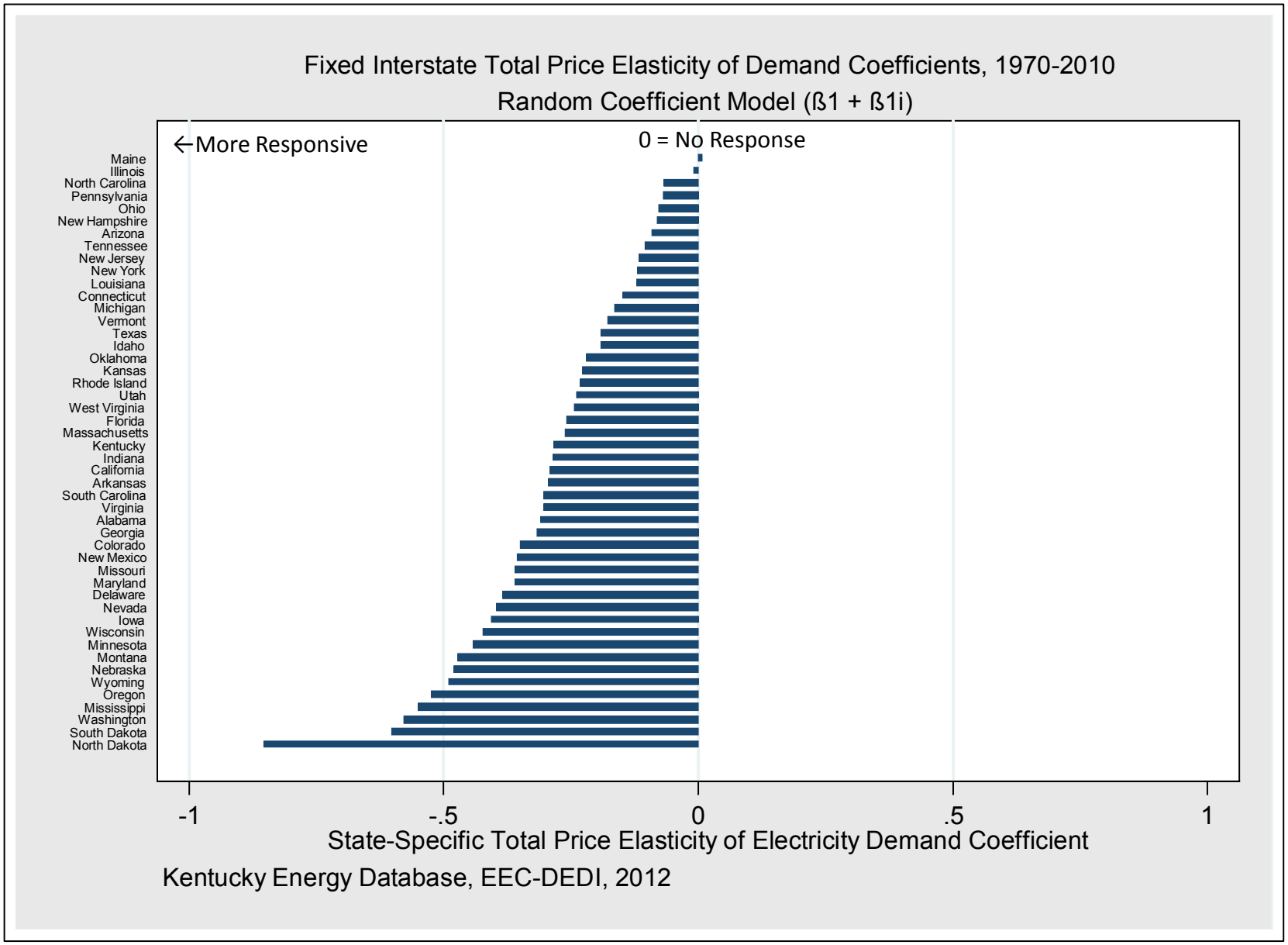
By allowing both β_{1i} *and* α_i to vary between units, the resulting historical electricity demand forecasts are significantly improved versus the inclusion of α_i alone. While this model produced the most-accurate historical predictions of the models evaluated here, it may also be the most vulnerable to over-specification of future demand in the event that the environmental or economic factors of a given state were to undergo a major transformation. When such a transformation is anticipated, a national model should be seriously considered.

Although this model does allow for spatial heterogeneity of demand and price elasticity, the resulting forecast may still also be biased for numerous reasons, including the temporal homogeneity that has been imposed on the coefficients.

An interactive spreadsheet version of this model is available for download at:
http://energy.ky.gov/Programs/Data%20Analysis%20%20Electricity%20Model/Residential_Electricity_Demand_Model.zip

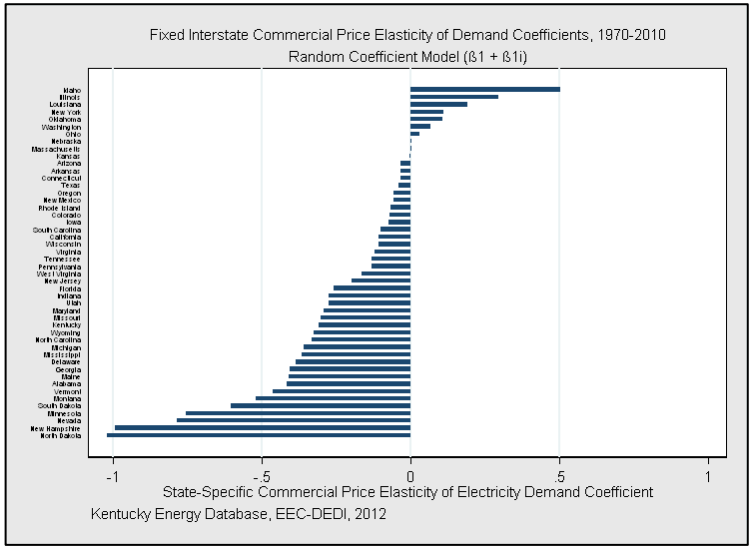
(Hsiao, Cheng et al. 1989)
(Hsiao, Cheng et al. 2004)

RCM: State-Specific PED ($\beta_1+\beta_{1i}$)

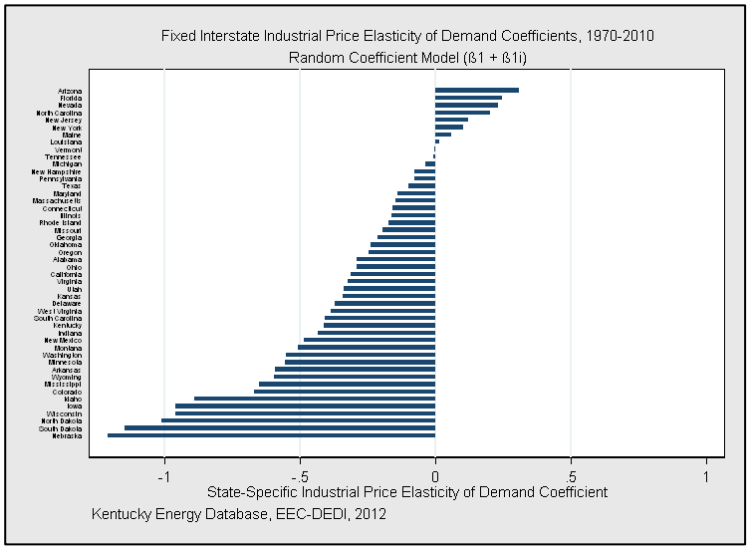


RCM: State-Specific PED ($\beta_1+\beta_{1i}$)

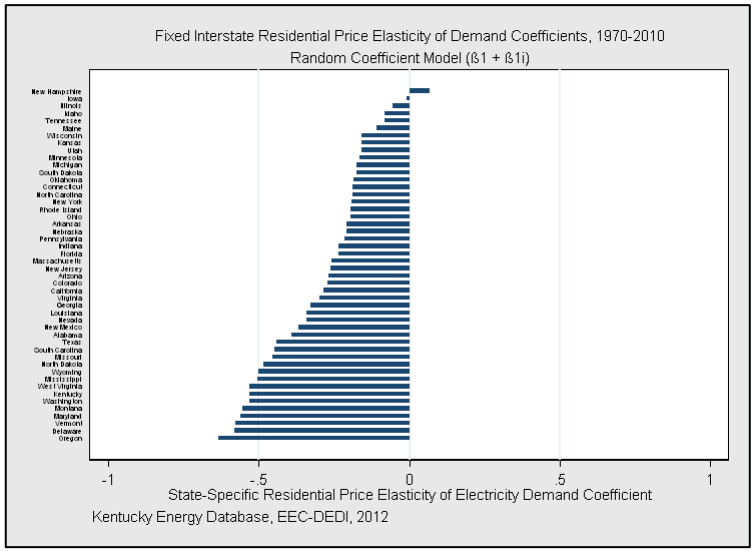
Commercial



Industrial



Residential



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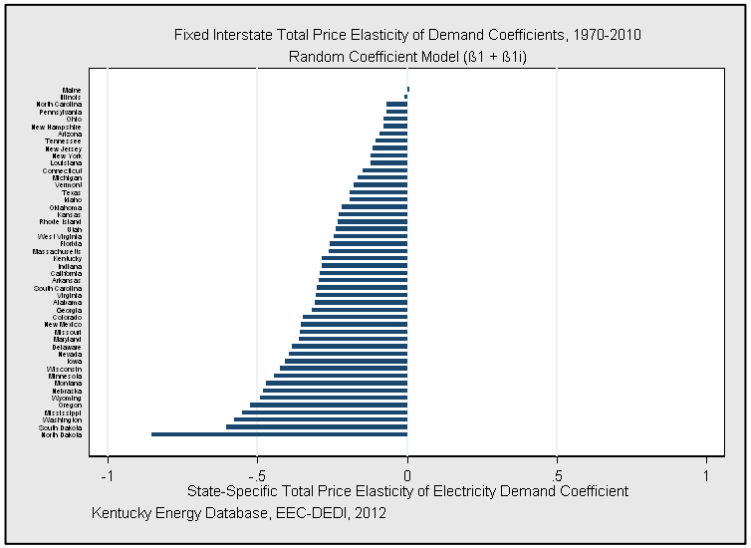
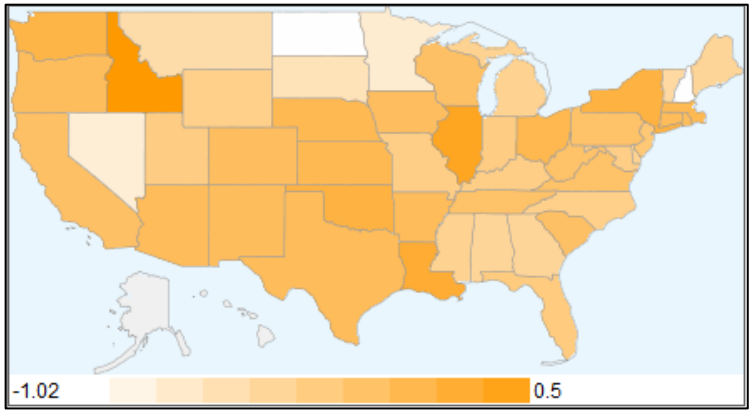


RCM: State-Specific PED ($\beta_1+\beta_{1i}$)

Commercial



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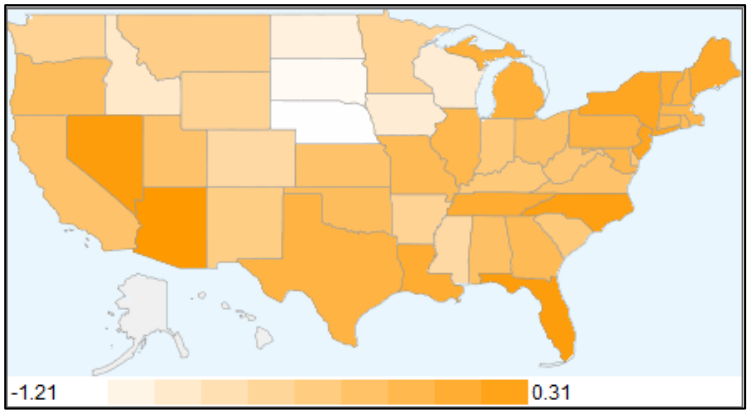

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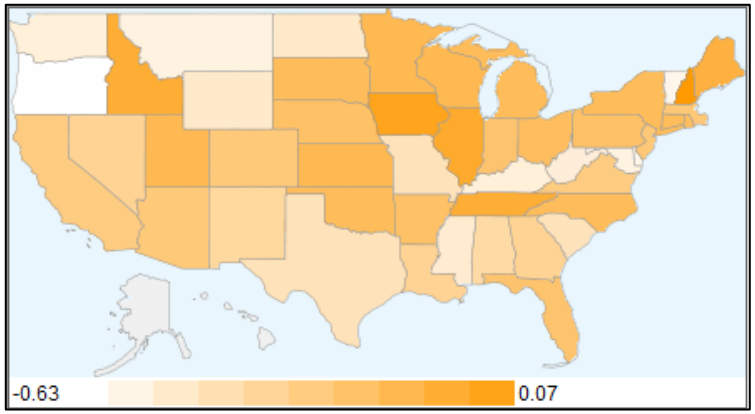

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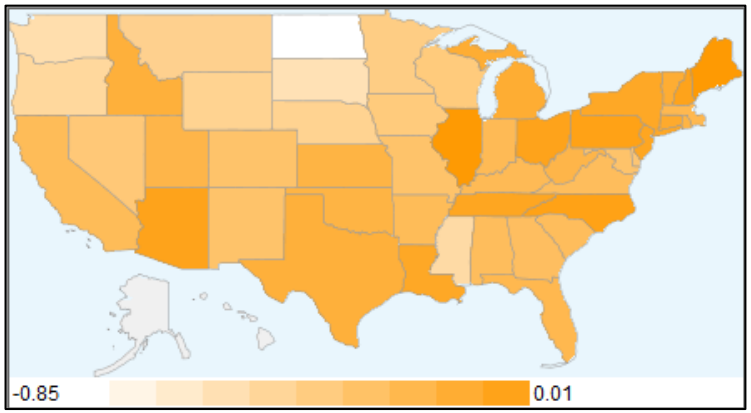


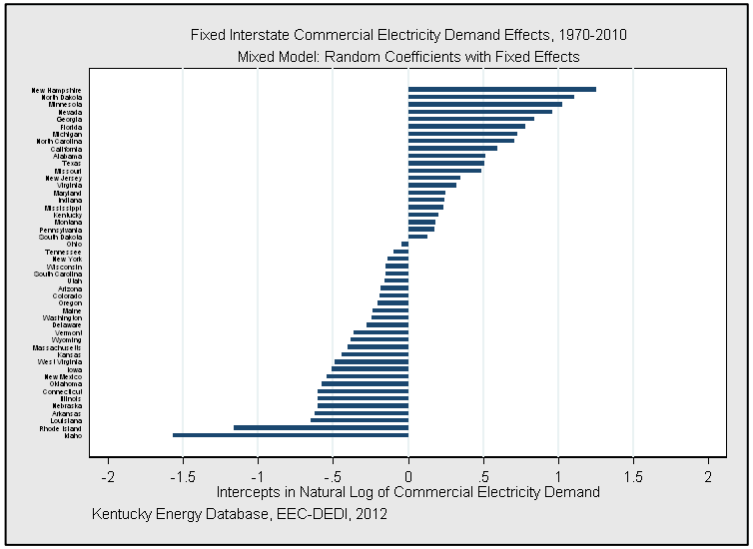
RCM: State-Specific PED ($\beta_1+\beta_{1i}$)

Long-Term Electricity Price Elasticity of Demand Coefficients by State and Sector									
State	**Total**	**Commercial**	**Industrial**	**Residential**	**State**	**Total**	**Commercial**	**Industrial**	**Residential**
Alabama	-0.3087599	-0.4170632	-0.2898364	-0.3913915	**Nebraska**	-0.4799013	0.0030113†	-1.209535	-0.2098557
Arizona	-0.0910499	-0.0332196	0.3085556†	-0.2669618	**Nevada**	-0.3962507	-0.7877516	0.2327869†	-0.3419631
Arkansas	-0.294216	-0.0340935	-0.5910203	-0.2092012	**New Hampshire**	-0.0800011	-0.9956621	-0.0770185	0.0692813†
California	-0.291176	-0.109461	-0.3144251	-0.2843532	**New Jersey**	-0.116318	-0.2005763	0.1202775†	-0.261656
Colorado	-0.3490135	-0.0711637	-0.6693799	-0.2704003	**New Mexico**	-0.3556828	-0.0574336	-0.4845073	-0.367289
Connecticut	-0.1477934	-0.035629	-0.1596523	-0.1881123	**New York**	-0.1202262	0.1100872†	0.1025288†	-0.1924082
Delaware	-0.3842452	-0.3888806	-0.3729327	-0.5807927	**North Carolina**	-0.0676994	-0.3335291	0.2007156†	-0.1898983
Florida	-0.2592883	-0.25978	0.2453772†	-0.2356396	**North Dakota**	-0.853114	-1.022121	-1.012547	-0.4857046
Georgia	-0.317026	-0.4086802	-0.2127883	-0.3283424	**Ohio**	-0.078168	0.0275787†	-0.291396	-0.1964576
Idaho	-0.1911237	0.5026617†	-0.8907084	-0.081678	**Oklahoma**	-0.219181	0.1064914†	-0.2377331	-0.185915
Illinois	-0.0096779	0.2946814†	-0.1610061	-0.0548459	**Oregon**	-0.524242	-0.0572237	-0.2450106	-0.6343877
Indiana	-0.2849264	-0.2752537	-0.4330406	-0.2335875	**Pennsylvania**	-0.0690976	-0.1317105	-0.077236	-0.2156862
Iowa	-0.4066845	-0.0738286	-0.9601065	-0.010225	**Rhode Island**	-0.2318595	-0.0691067	-0.173886	-0.1956412
Kansas	-0.2269027	-0.0042212	-0.3412006	-0.1581247	**South Carolina**	-0.302727	-0.1024556	-0.4086682	-0.4487473
Kentucky	-0.2839586	-0.3087448	-0.4125502	-0.5318559	**South Dakota**	-0.6029056	-0.6071295	-1.147973	-0.1754516
Louisiana	-0.1208637	0.1907391†	0.0144001†	-0.3401106	**Tennessee**	-0.1040273	-0.1306986	-0.0081116	-0.0823142
Maine	0.0089881†	-0.4105483	0.0569597†	-0.1090858	**Texas**	-0.1905553	-0.0418771	-0.0989334	-0.4410236
Maryland	-0.3597877	-0.2937967	-0.1401375	-0.559975	**Utah**	-0.2390822	-0.2763781	-0.3391246	-0.1594099
Massachusetts	-0.2610833	0.0003632†	-0.1472141	-0.2580441	**Vermont**	-0.1779001	-0.4649802	-0.0023459	-0.5776038
Michigan	-0.1641571	-0.3614348	-0.0356216	-0.1739007	**Virginia**	-0.3036528	-0.1222396	-0.3222929	-0.2990341
Minnesota	-0.4429693	-0.7580849	-0.5571533	-0.1640091	**Washington**	-0.5779196	0.0655186†	-0.5501498	-0.5318817
Mississippi	-0.5506878	-0.3685589	-0.6523803	-0.5048864	**West Virginia**	-0.2432454	-0.1661688	-0.3872817	-0.5294507
Missouri	-0.3596196	-0.3028014	-0.1963625	-0.4552325	**Wisconsin**	-0.4228646	-0.1098113	-0.9604038	-0.1572106
Montana	-0.4716608	-0.5224722	-0.5083855	-0.553391	**Wyoming**	-0.4907126	-0.3256405	-0.5958909	-0.4994614

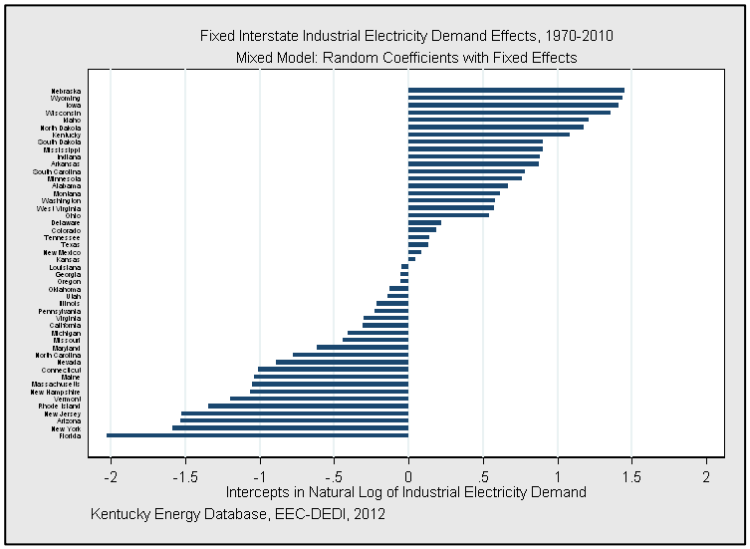
† *Positive price elasticity of demand coefficients are irrational, and should be interpreted as zero—that the model failed to detect any effect for price.*

RCM: Fixed Effects for Demand (α_i)

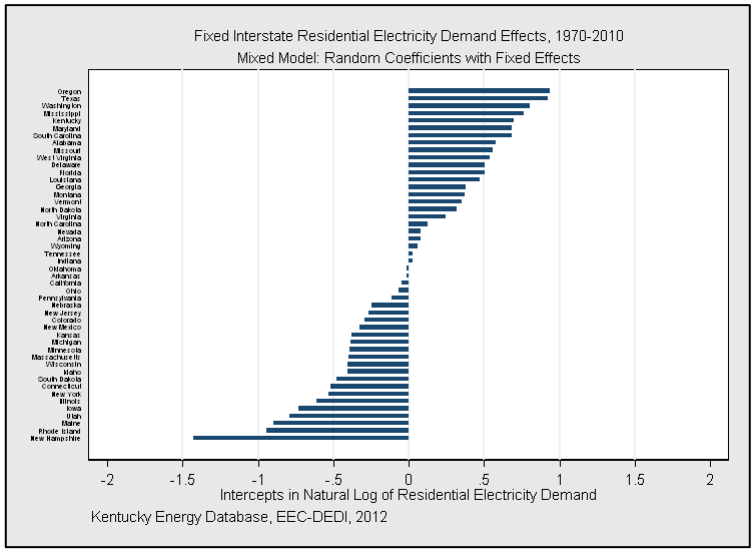
Commercial



Industrial



Residential



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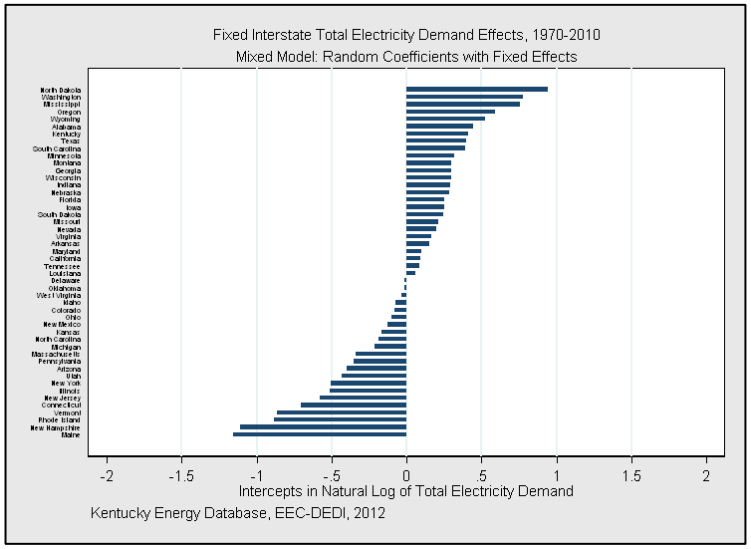
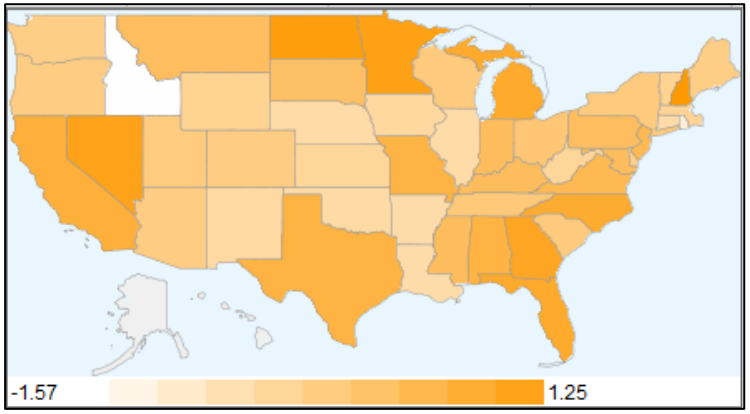


States where the natural log of electricity demand is higher or lower than expected even after controlling for all of the independent variables.

RCM: Fixed Effects for Demand (α_i)

Commercial



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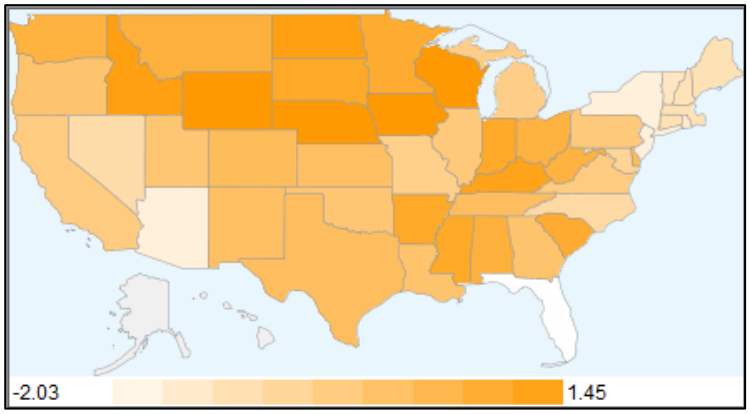
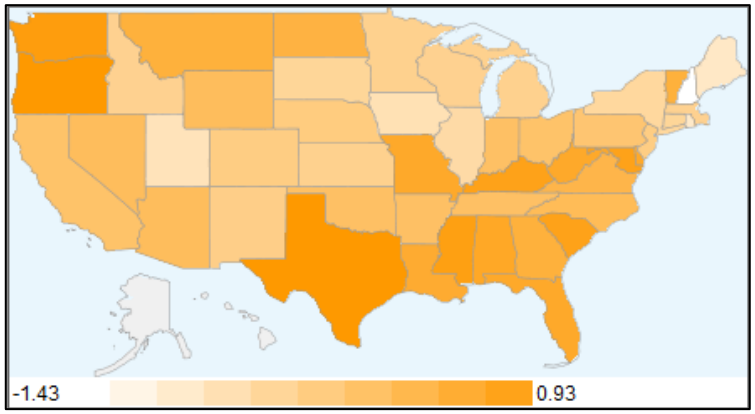


Residential



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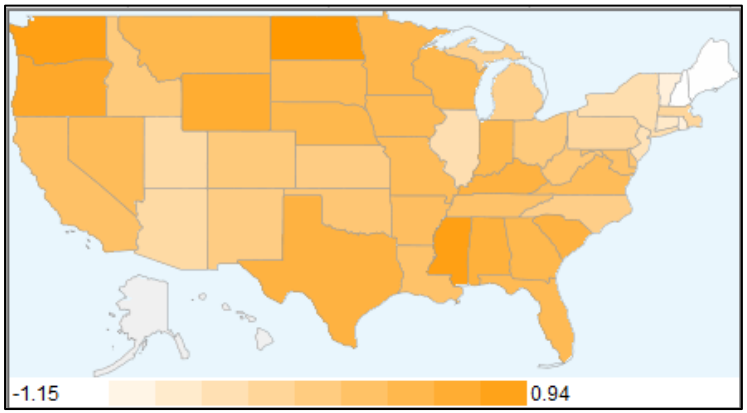
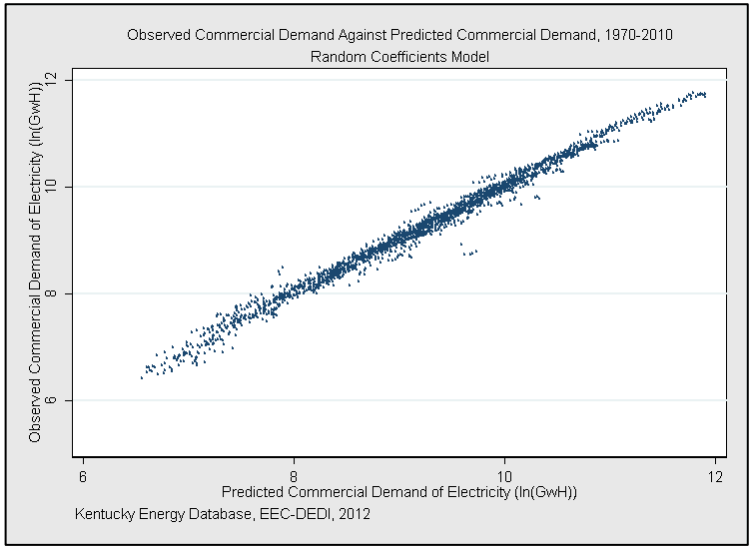


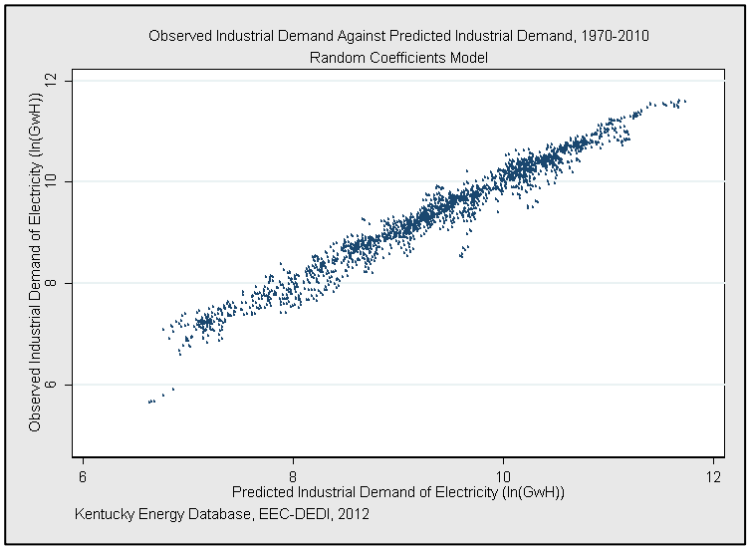
States where the natural log of electricity demand is higher or lower than expected even after controlling for all of the independent variables.

RCM Model –Observed vs. Predicted

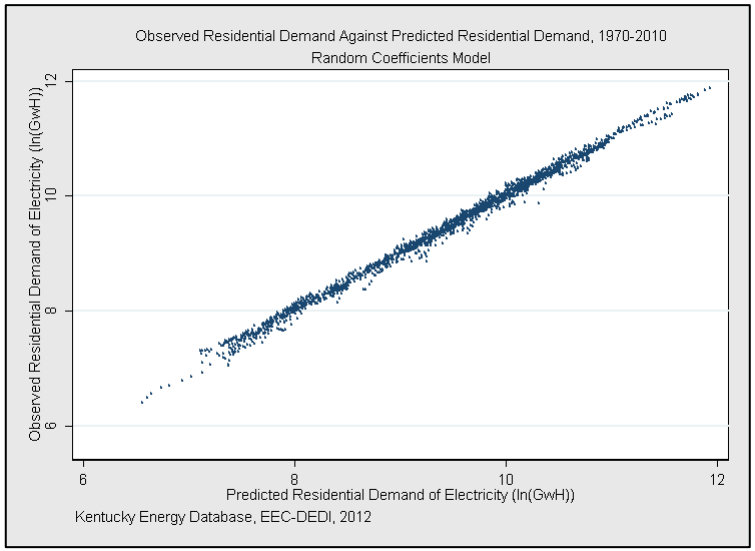
Commercial



Industrial



Residential



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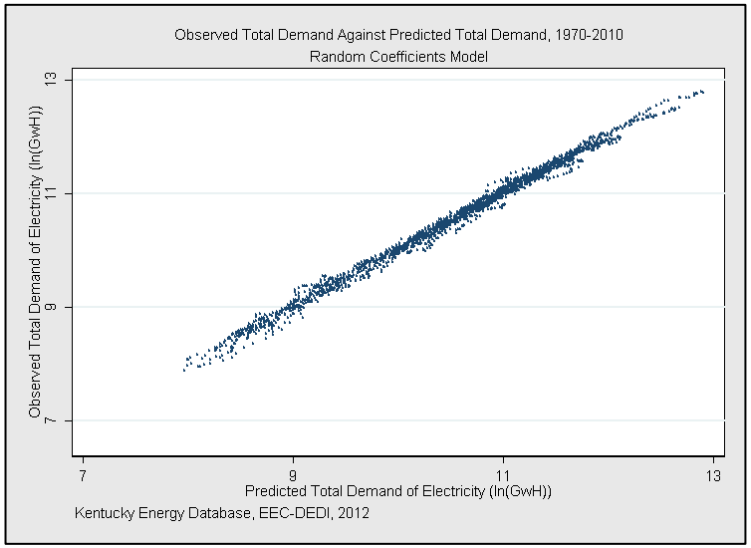
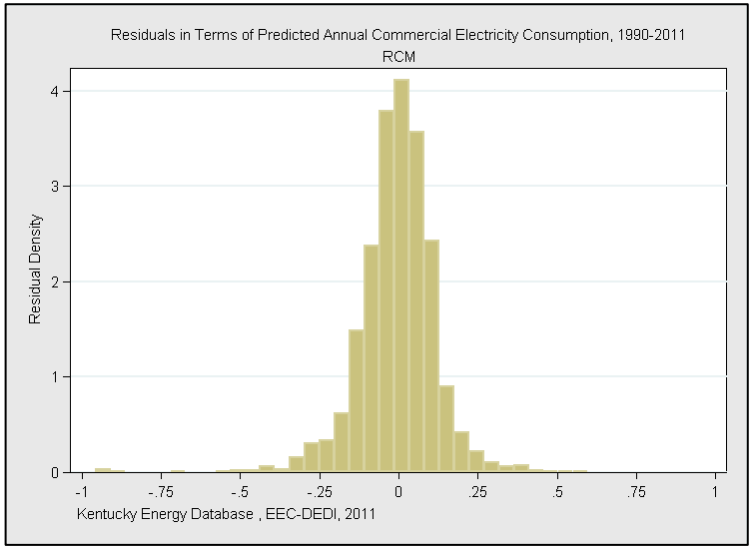


RCM Model – Residual Histograms

Commercial



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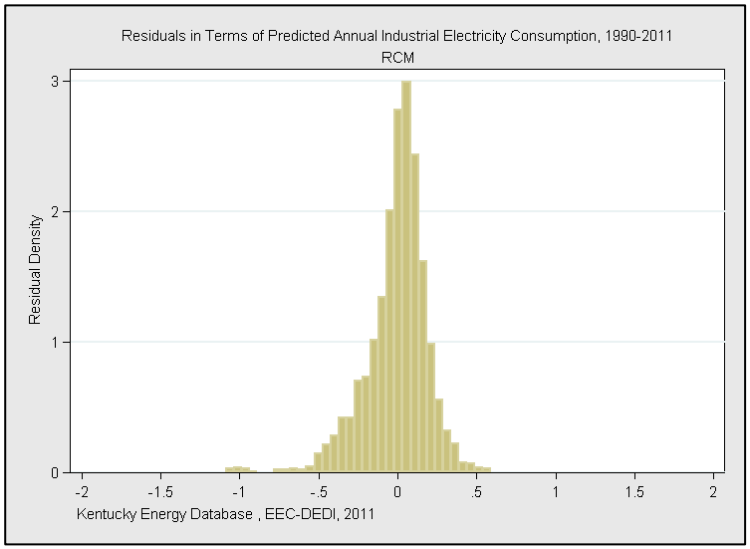
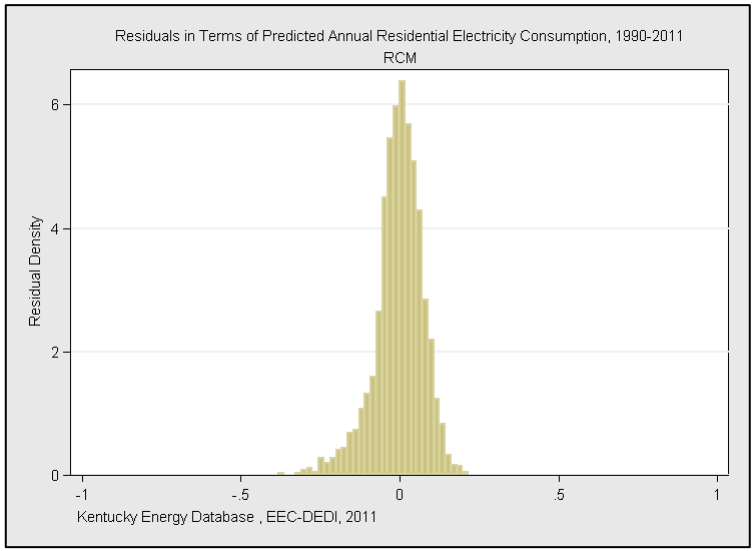


Residential



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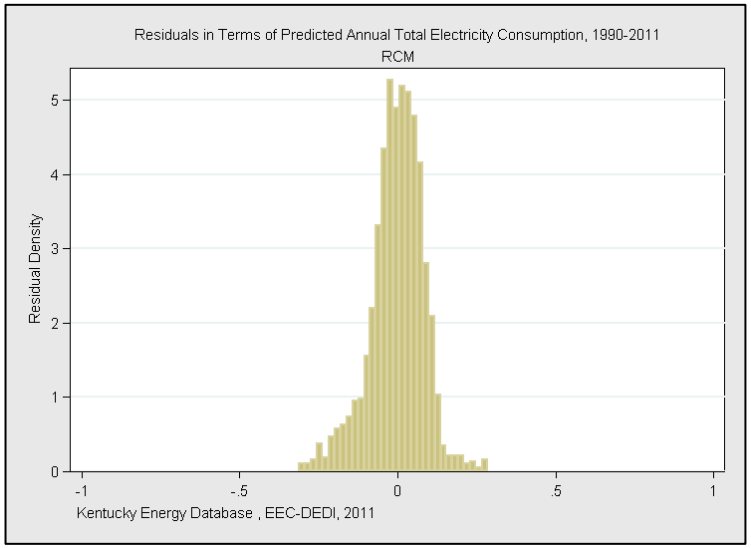
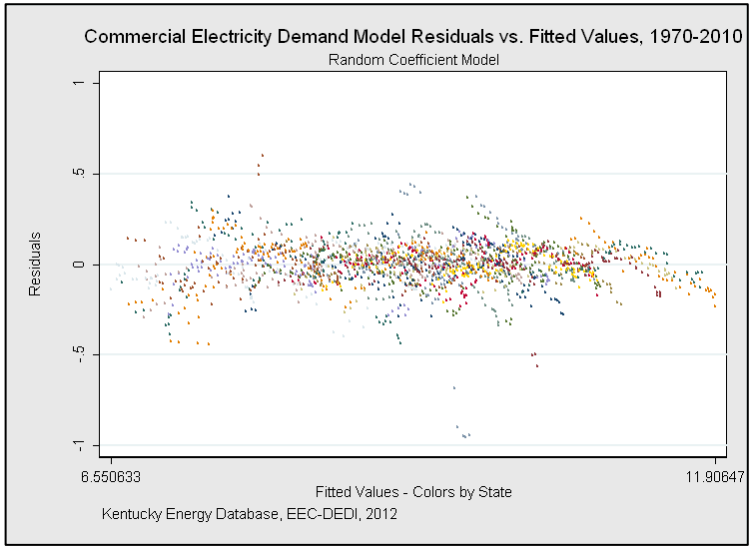


RCM Model – Residual Scatter Plots

Commercial



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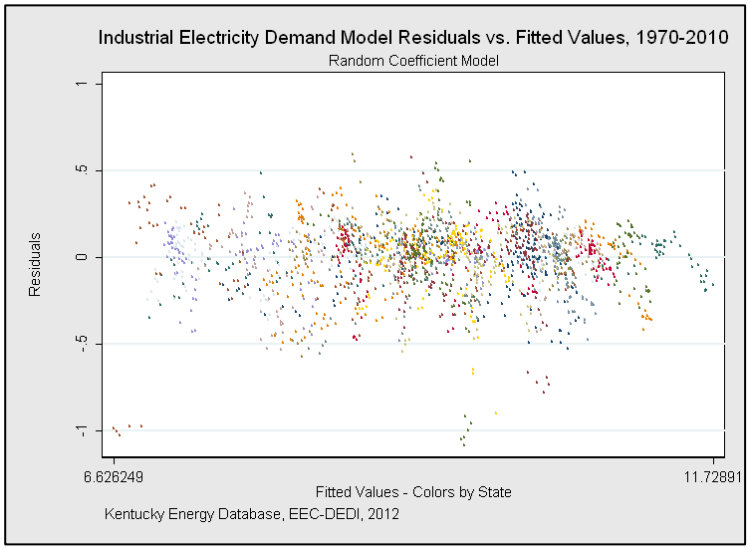

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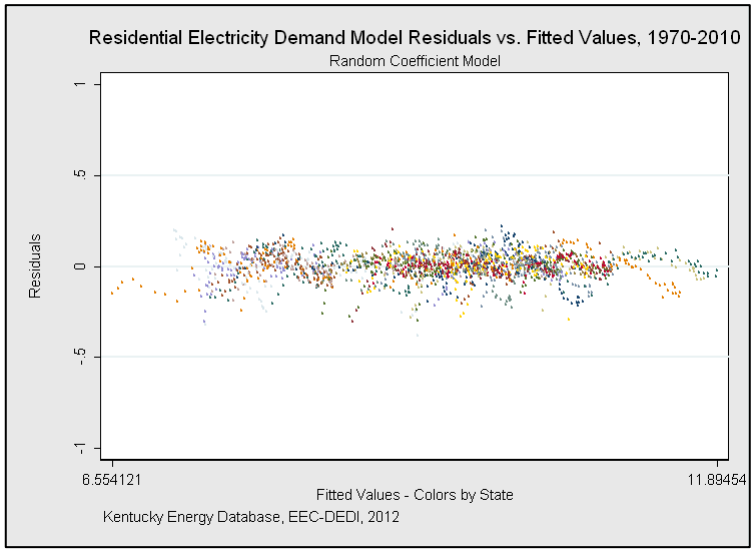

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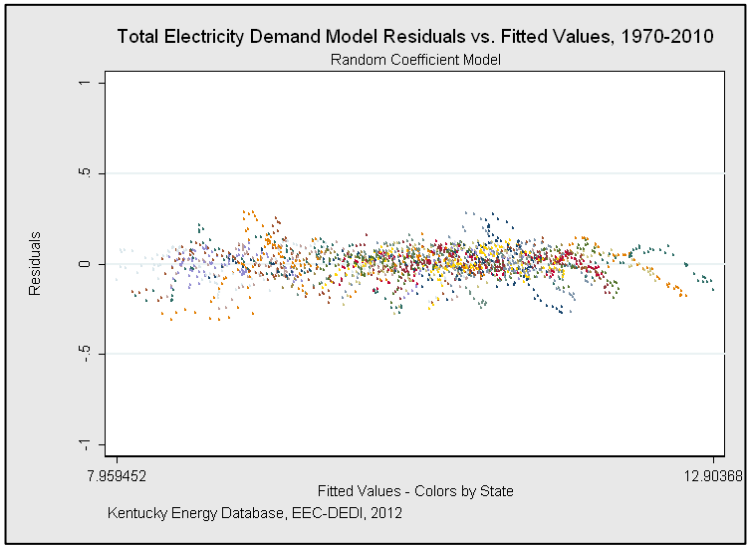
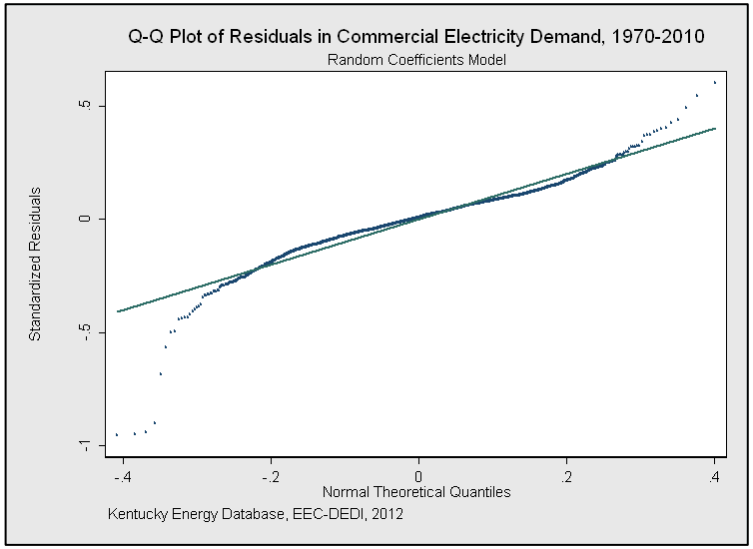


**Homoscedastic & Unbiased*

RCM Model – Q-Q Plots

Commercial



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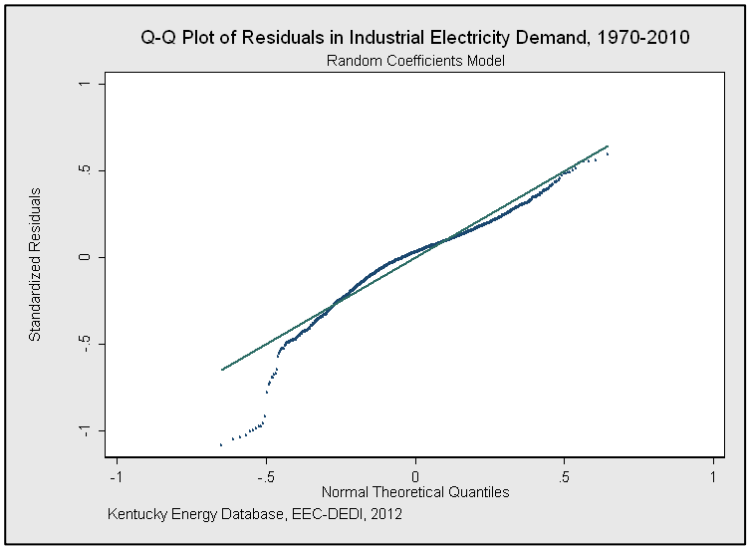
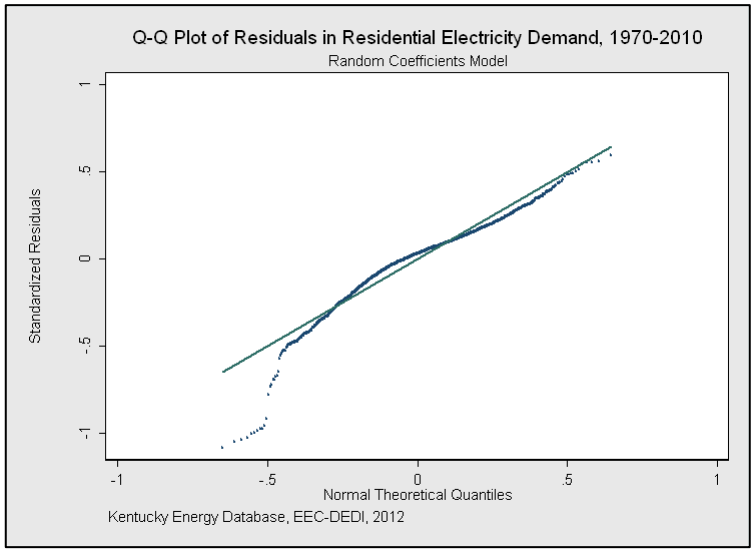


Residential



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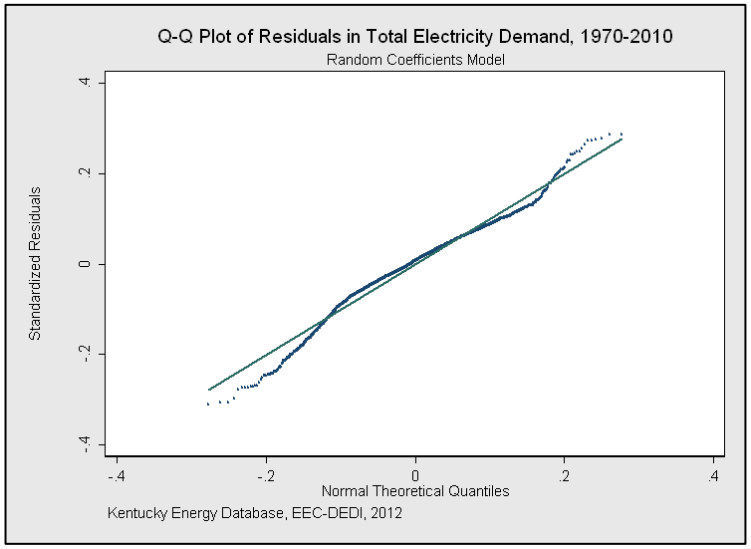


Varying Coefficient Model

VCM Model

$$Y_{it} = \beta_{0t} + \sum_{j=1}^{k-1} \beta_{jt} X_{jit} + \varepsilon_{it}$$

$$i = 1, \ldots, 48 \ (state)$$

$$t = 1970, \ldots, 2010 \ (year)$$

$$k = 8$$

Where *i* and *t* index states and years, such that y_{it} is the dependent variable of interest, electricity consumption by economic sector, in state *i* in year *t*, β_{0t} is the time varying *y* intercept across all states, *X* is a *k* x 1 vector of explanatory variables, $\beta_{jt}X_{jit}$ is the product of the observation for each independent variable *j* through *k* for state *i* in year *t* and the coefficient of *X in year t*, *k* is the total number of included independent variables, and ε_{it} are the residuals, and where $\varepsilon_{it} \sim N(0, \sigma^2)$, or are approximately normally distributed with a mean of zero.
(Hastie, Trevor & Tibshirani, Robert. 1993)

VCM Model

Given the multidimensional nature of panel data, which has both cross-sections (*i*), and a time series dimension (*t*), after quantifying how elasticity coefficients may vary across cross-sections (states), this study then estimated the degree to which PED coefficients vary across time.

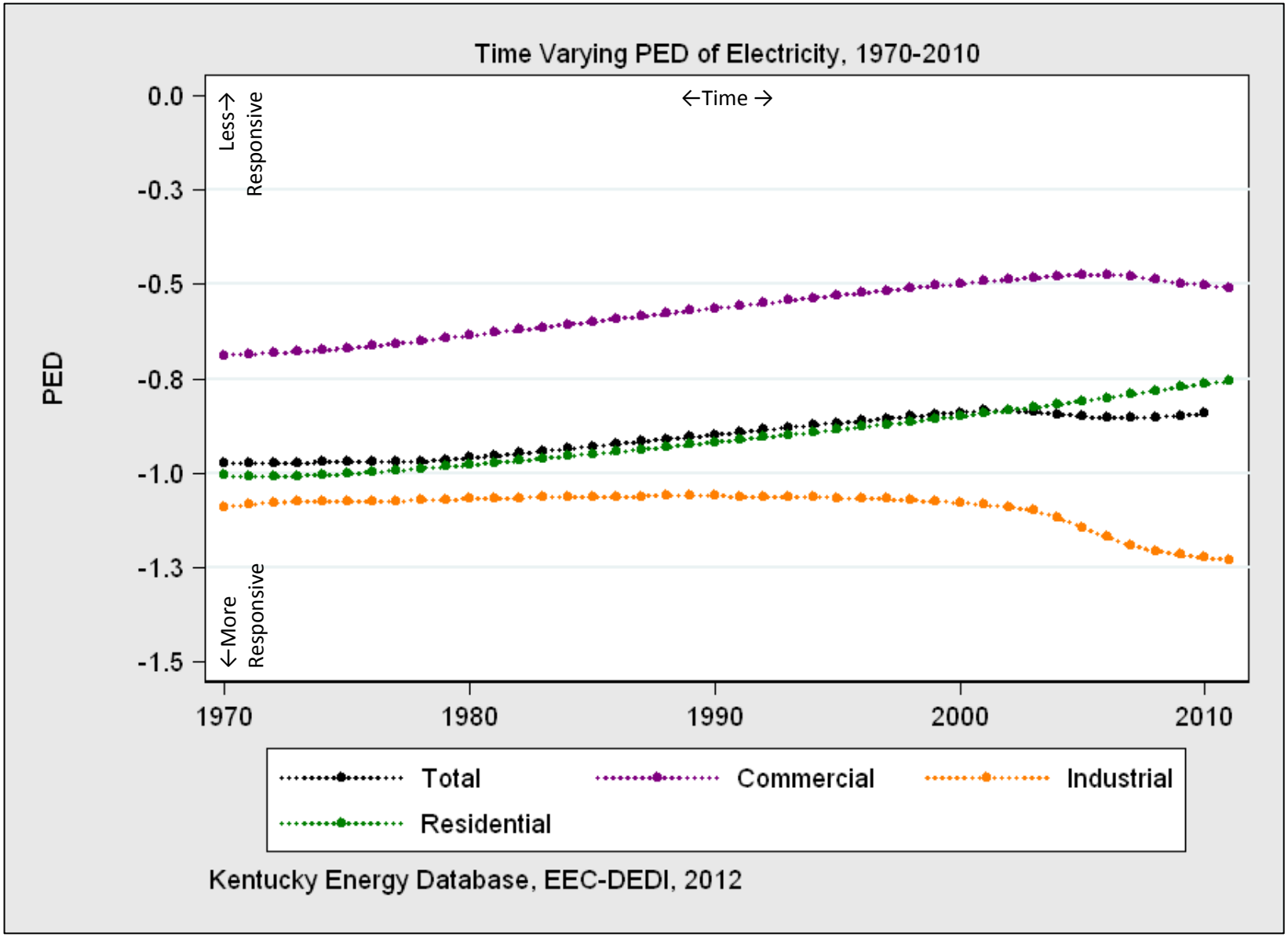
The Varying Coefficient Model, referred to as VCM, builds upon Ordinary Least Squares (OLS) regression by isolating the time-varying difference in electricity price elasticity and other effects of demand nationally.

Unlike the Fixed Effects or Random Coefficient Model, there are no fixed effects (α_i) and the coefficient for price (β_{1i}) is not permitted to vary from state to state.

With no state-specific intercept and homogenous PED coefficients again being imposed across states, the Varying Coefficient Model, as we have specified it, did not perform as well at predicting electricity demand as either the Fixed Effects or Random Coefficient Model. This suggests that the time variance of price elasticity of demand may be less significant than regional differences.

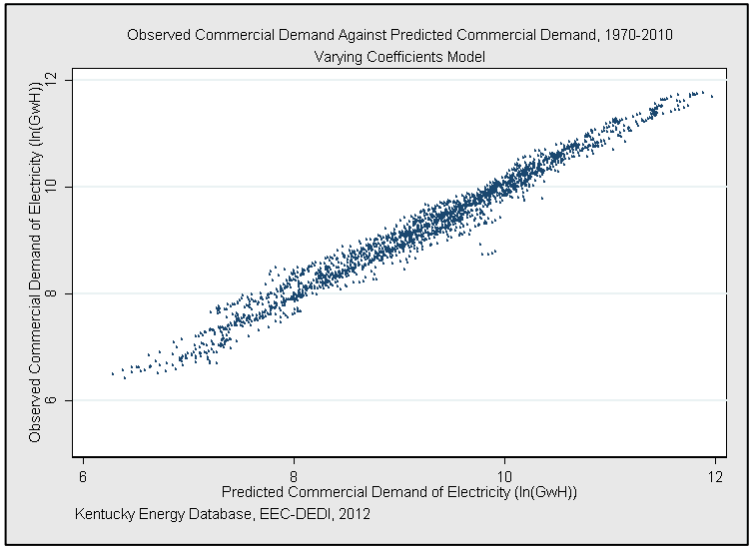
(Hastie, Trevor & Tibshirani, Robert. 1993)

Time-Varying PED Coefficients



VCM Model – Observed vs. Predicted

Commercial



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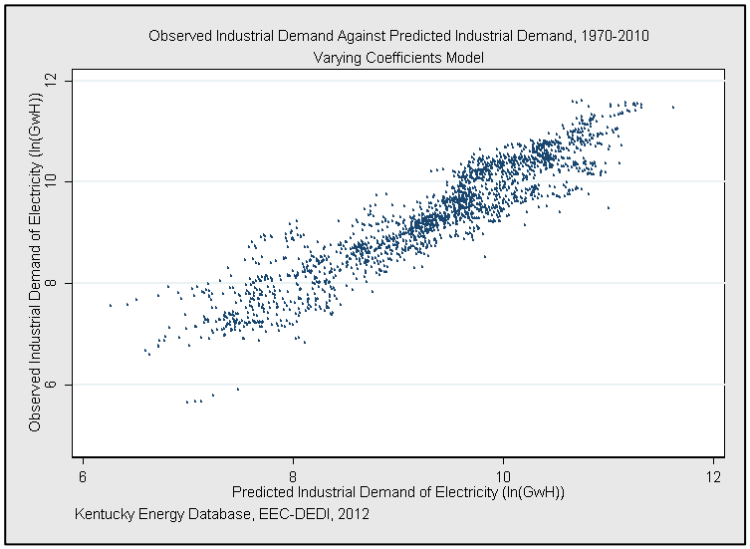
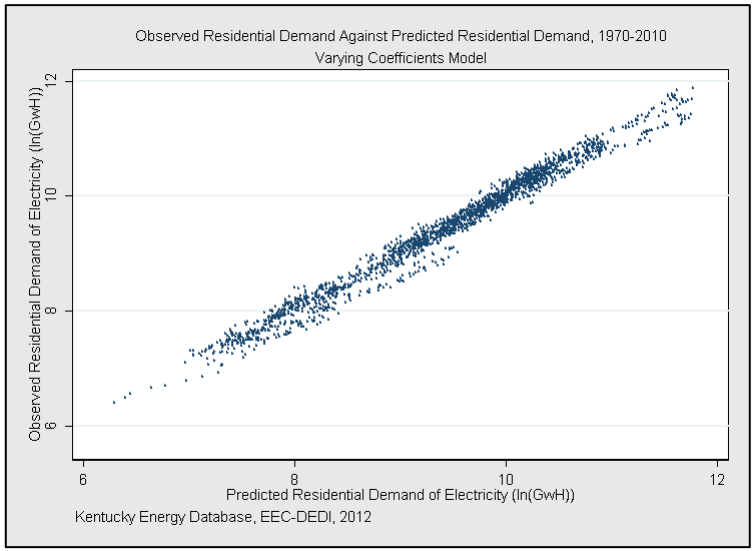


Residential



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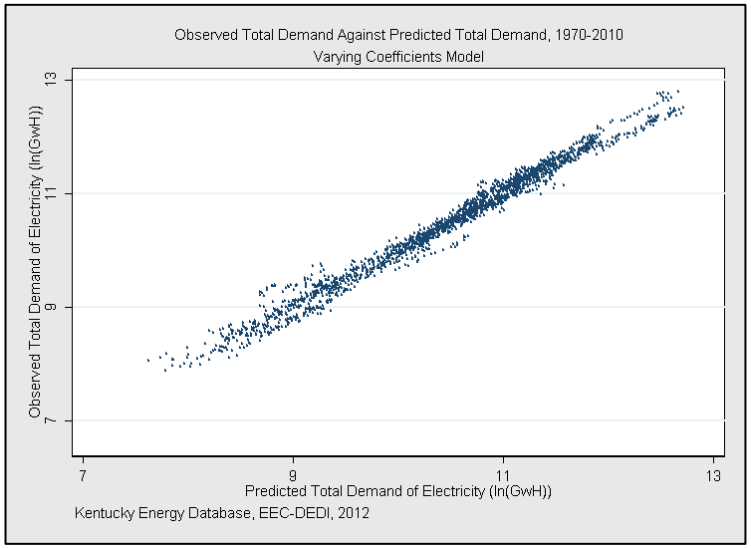
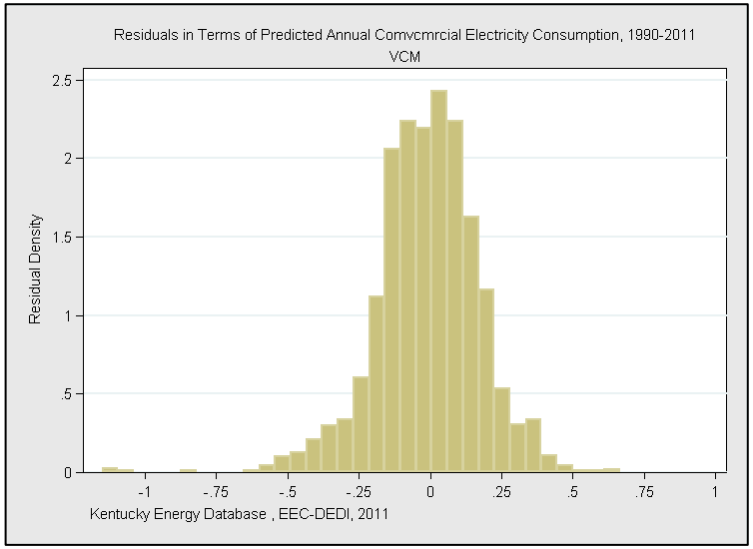
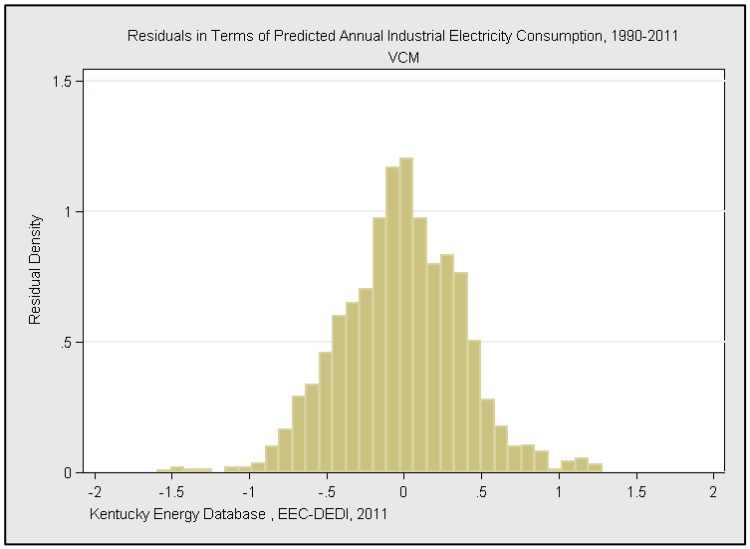


VCM Model – Residual Histogram

Commercial



Industrial



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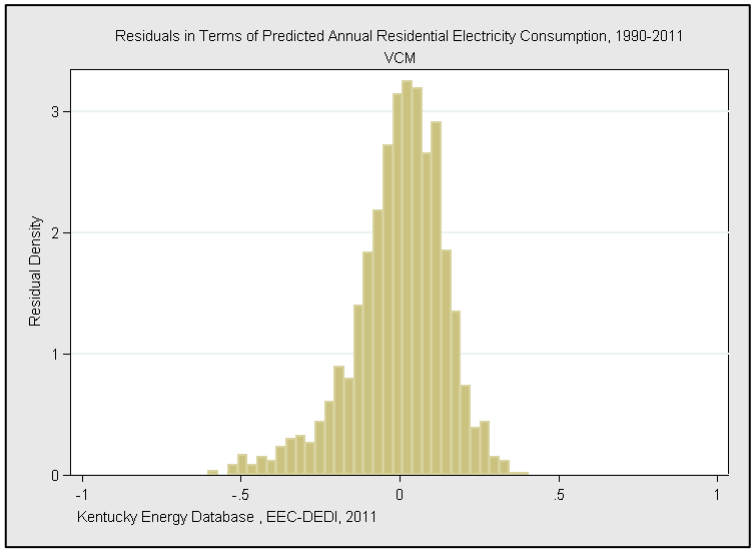

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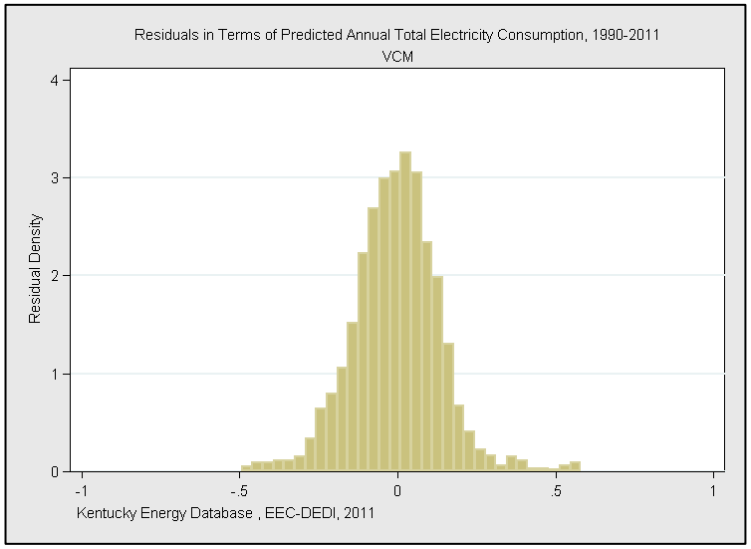
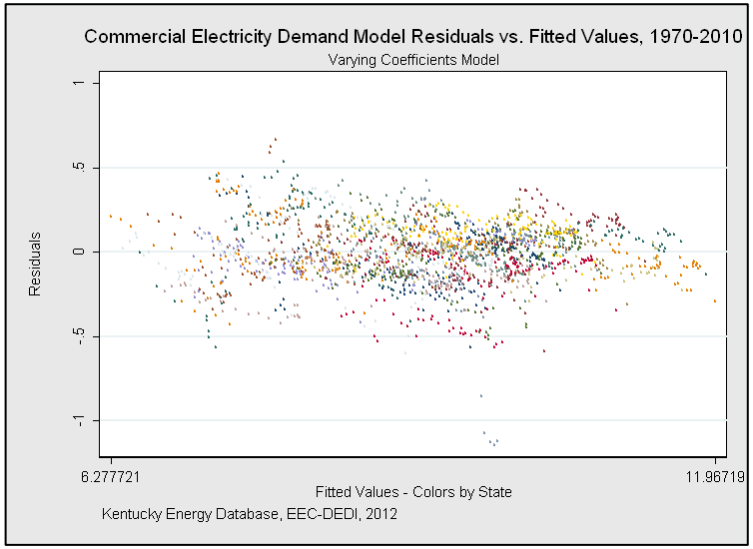


VCM Model – Residual Scatter Plots

Commercial



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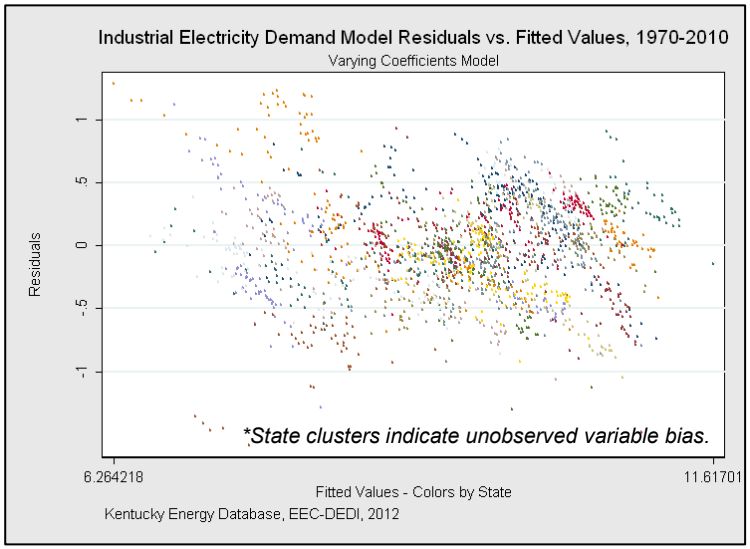

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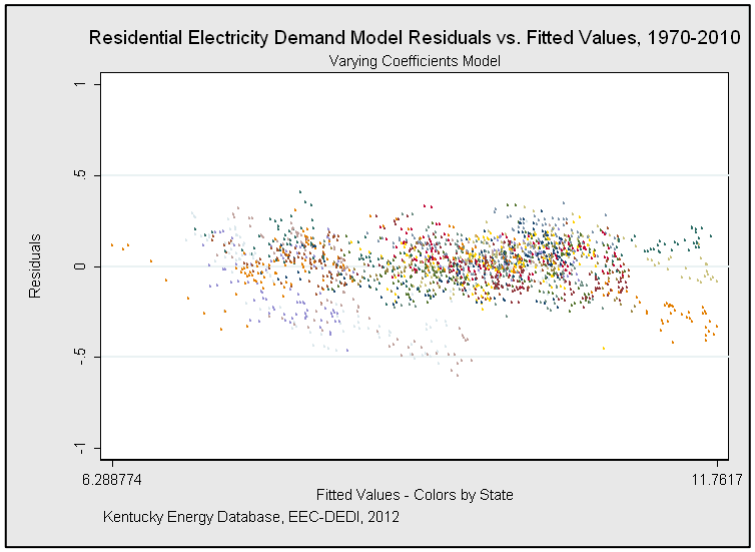

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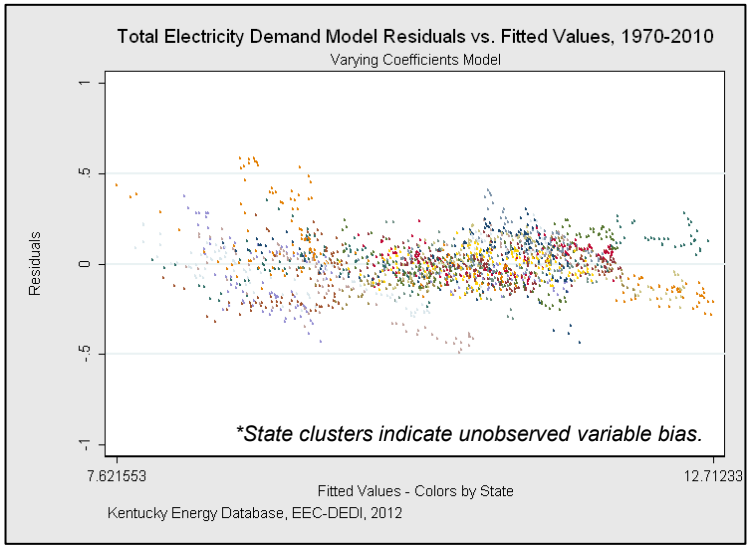
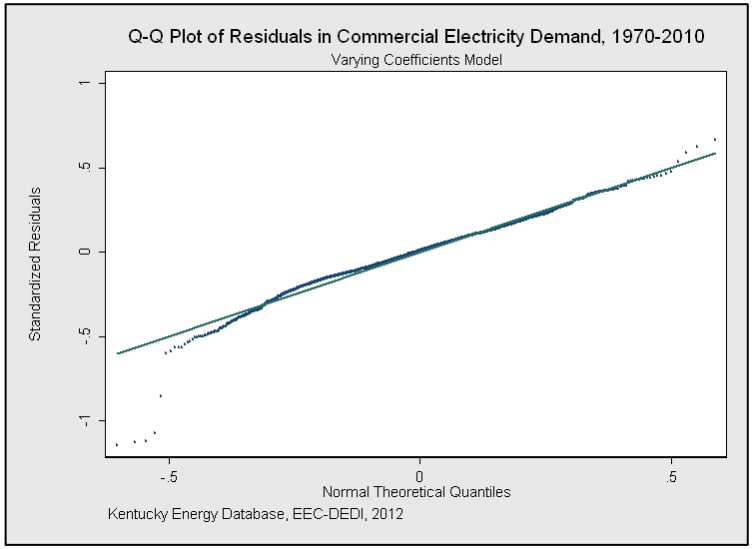


**Heteroscedasticity Identified*

VCM Model – Q-Q Plots

Commercial



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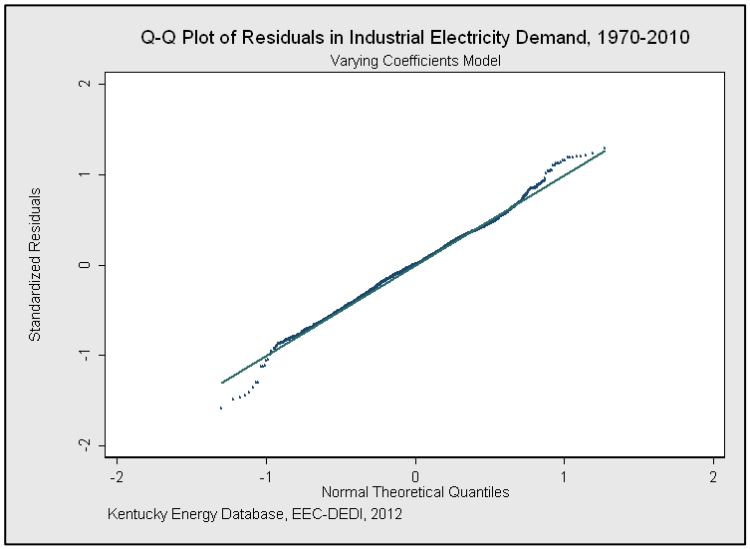

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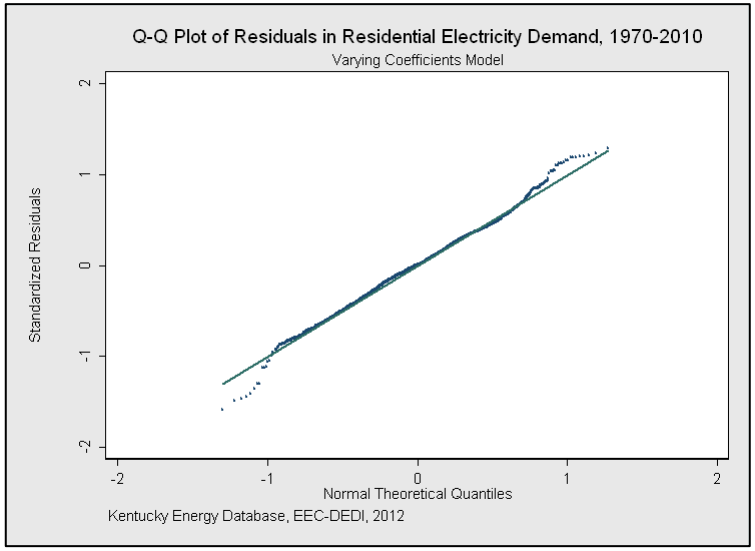

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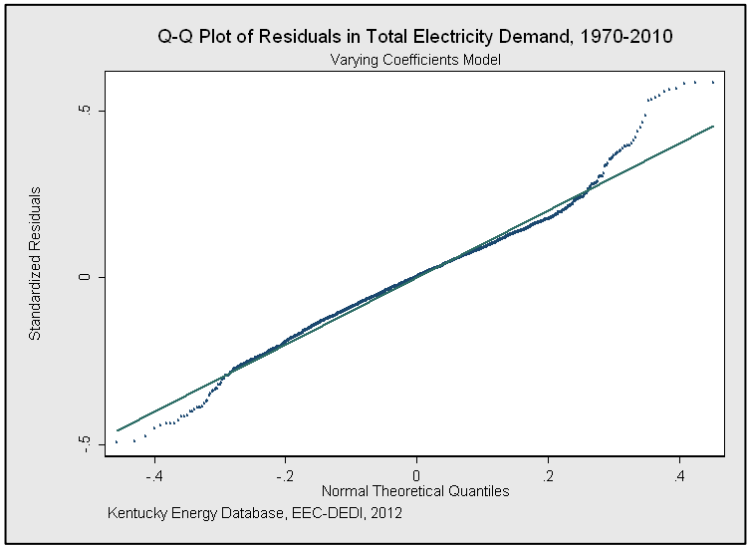


Model Comparison

Total Demand Models

Logged Variables	*Heterogeneous*		*Homogeneous*	
	RCM Model	**FE Model**	**VCM Model**	**OLS Model**
Real Total Electricity Price	-0.288***	-0.320***	-0.321***	-0.833***
	(0.03)	(0.02)	(0.02)	(0.02)
Cooling Degree Days	0.002	-0.014	-0.013	0.077***
	(0.01)	(0.01)	(0.01)	(0.01)
Heating Degree Days	-0.018	0.015	0.017	-0.112***
	(0.03)	(0.03)	(0.03)	(0.01)
Population	0.783***	0.709***	0.716***	0.959***
	(0.02)	(0.02)	(0.02)	(0.00)
Year	17.328***	17.497***	9.593**	11.466***
	(1.45)	(1.46)	(3.29)	(1.34)
Real Total Natural Gas Price	0.136***	0.147***	0.153***	0.214***
	(0.01)	(0.01)	(0.01)	(0.01)
Real Per Capita Personal Income	0.288***	0.324***	0.310***	-0.008
	(0.04)	(0.04)	(0.04)	(0.03)
Constant	-135.359***	-136.008***	-75.957**	-89.012***
	(10.51)	(10.63)	(24.76)	(9.95)
R-Squared		0.92370		0.97340

Coefficients with Standard Errors in Parentheses

* p<0.05, ** p<0.01, *** p<0.001

The state-specific fixed effects and random coefficients, as well as the homogenous time varying components are not listed above.

Commercial Demand Models

Logged Variables	*Heterogeneous*		*Homogeneous*	
	RCM Model	**FE Model**	**VCM Model**	**OLS Model**
Real Commercial Electricity Price	-0.211***	-0.205***	-0.214***	-0.536***
	(0.05)	(0.02)	(0.02)	(0.02)
Cooling Degree Days	-0.003	-0.008	-0.006	0.030**
	(0.02)	(0.02)	(0.02)	(0.01)
Heating Degree Days	-0.090*	-0.044	-0.037	-0.173***
	(0.04)	(0.04)	(0.04)	(0.01)
Population	0.522***	0.458***	0.476***	0.954***
	(0.03)	(0.03)	(0.03)	(0.01)
Year	38.911***	38.593***	24.006***	15.991***
	(2.16)	(2.21)	(5.49)	(1.69)
Real Commercial Natural Gas Price	0.111***	0.112***	0.120***	0.095***
	(0.01)	(0.01)	(0.01)	(0.02)
Real Per Capita Personal Income	0.372***	0.457***	0.433***	0.611***
	(0.06)	(0.06)	(0.06)	(0.04)
Constant	-296.951***	-294.830***	-184.104***	-130.521***
	(15.67)	(16.1)	(41.39)	(12.53)
R-Squared		0.85920		0.96800

Coefficients with Standard Errors in Parentheses

* p<0.05, ** p<0.01, *** p<0.001

The state-specific fixed effects and random coefficients, as well as the homogenous time varying components are not listed above.

Industrial Demand Models

Logged Variables	*Heterogeneous*		*Homogeneous*	
	RCM Model	**FE Model**	**VCM Model**	**OLS Model**
Real Industrial Electricity Price	-0.321***	-0.337***	1.021***	-0.943***
	(0.06)	(0.03)	(0.07)	(0.03)
Cooling Degree Days	-0.052*	-0.091**	-0.056*	0.123***
	(0.03)	(0.03)	(0.03)	(0.02)
Heating Degree Days	-0.077	-0.05	0.103	0.176***
	(0.06)	(0.07)	(0.06)	(0.03)
Population	0.886***	0.726***	0.732***	0.984***
	(0.04)	(0.05)	(0.04)	(0.01)
Year	-1.932	0.276	14.391***	18.711***
	(3.22)	(3.33)	(3.1)	(3.28)
Real Industrial Natural Gas Price	0.169***	0.190***	0.202***	0.299***
	(0.02)	(0.02)	(0.01)	(0.03)
Real Per Capita Personal Income	0.11	0.183	0.083	-1.013***
	(0.09)	(0.1)	(0.09)	(0.08)
Constant	10.906	-4.171	-111.882***	-138.065***
	(23.42)	(24.19)	(22.54)	(24.32)
R-Squared		0.75860		0.84260

Coefficients with Standard Errors in Parentheses

* p<0.05, ** p<0.01, *** p<0.001

The state-specific fixed effects and random coefficients, as well as the homogenous time varying components are not listed above.

Residential Demand Models

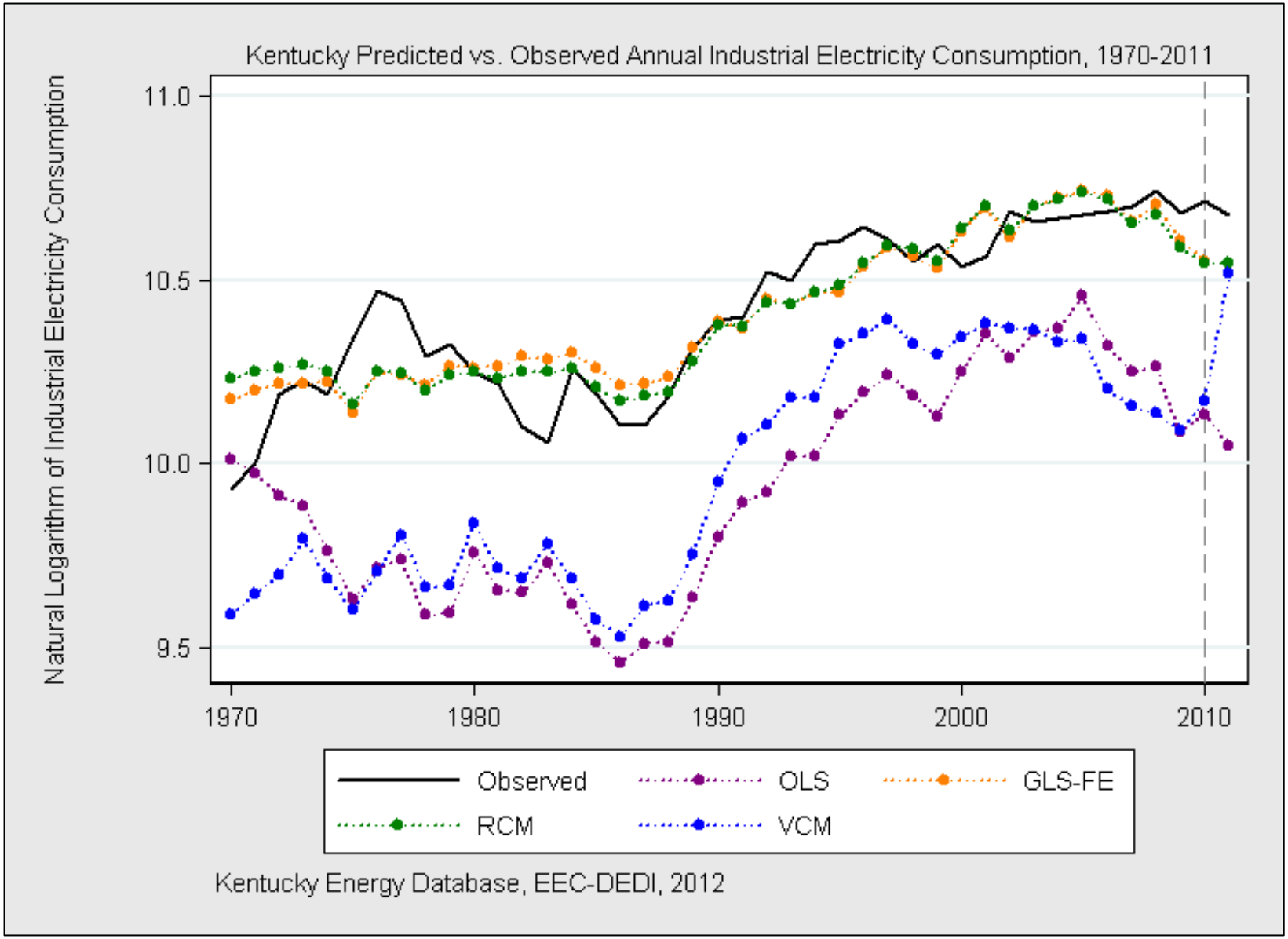
Logged Variables	*Heterogeneous*		*Homogeneous*	
	RCM Model	**FE Model**	**VCM Model**	**OLS Model**
Real Residential Electricity Price	-0.297***	-0.310***	-0.315***	-0.861***
	(0.03)	(0.02)	(0.02)	(0.02)
Cooling Degree Days	0.036***	0.033**	0.034**	0.118***
	(0.01)	(0.01)	(0.01)	(0.01)
Heating Degree Days	0.084***	0.112***	0.117***	-0.063***
	(0.02)	(0.03)	(0.03)	(0.01)
Population	0.821***	0.787***	0.805***	0.962***
	(0.02)	(0.02)	(0.02)	(0)
Year	20.903***	21.179***	4.884	11.001***
	(1.33)	(1.33)	(3.5)	(1.28)
Real Residential Natural Gas Price	0.183***	0.188***	0.194***	0.328***
	(0.01)	(0.01)	(0.01)	(0.02)
Real Per Capita Personal Income	0.160***	0.161***	0.145***	-0.086**
	(0.04)	(0.04)	(0.04)	(0.03)
Constant	-163.982***	-165.768***	-42.142	-86.562***
	(9.69)	(9.67)	(26.4)	(9.51)
R-Squared		0.93140		0.97690

Coefficients with Standard Errors in Parentheses

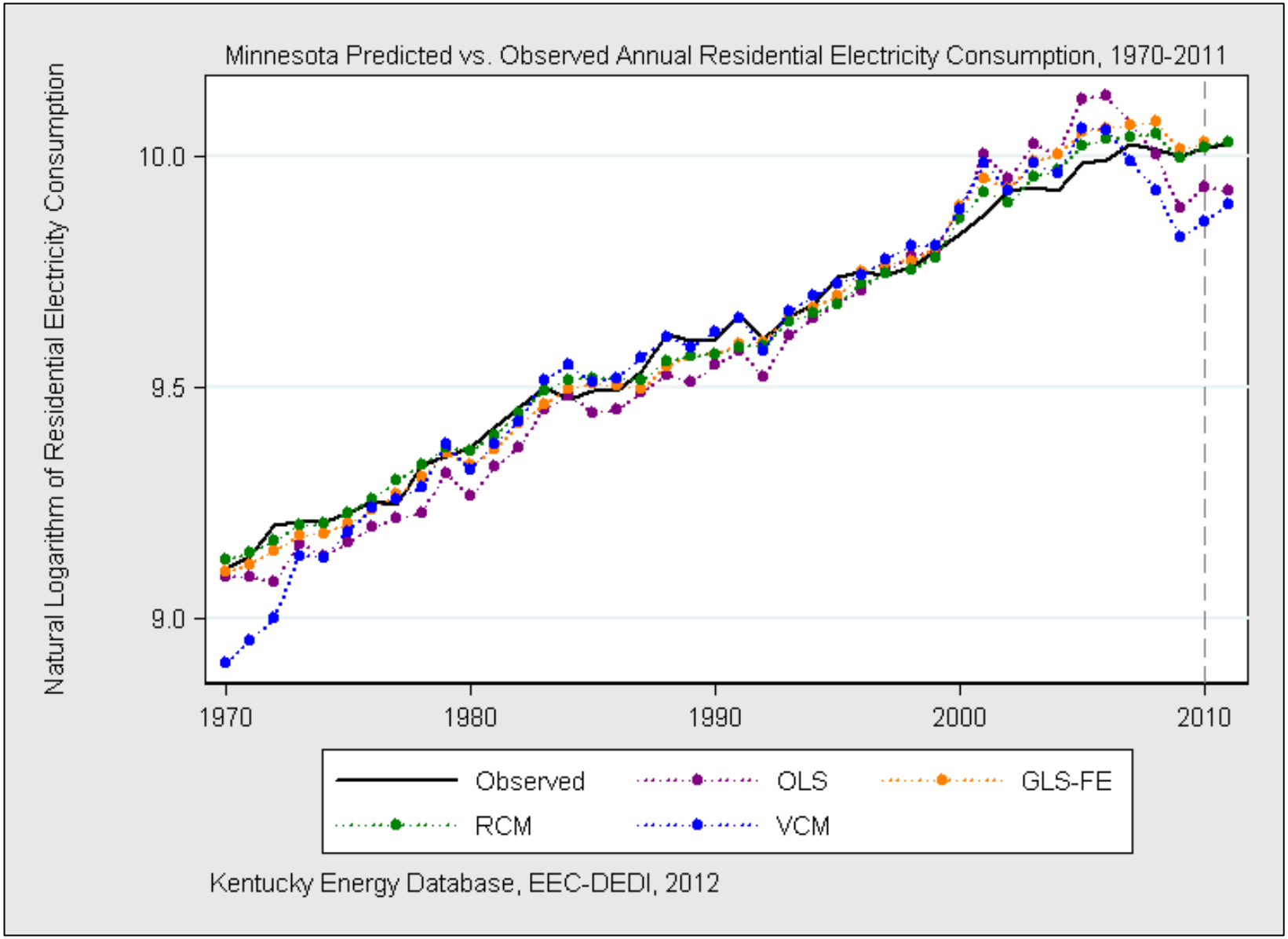
* p<0.05, ** p<0.01, *** p<0.001

The state-specific fixed effects and random coefficients, as well as the homogenous time varying components are not listed above.

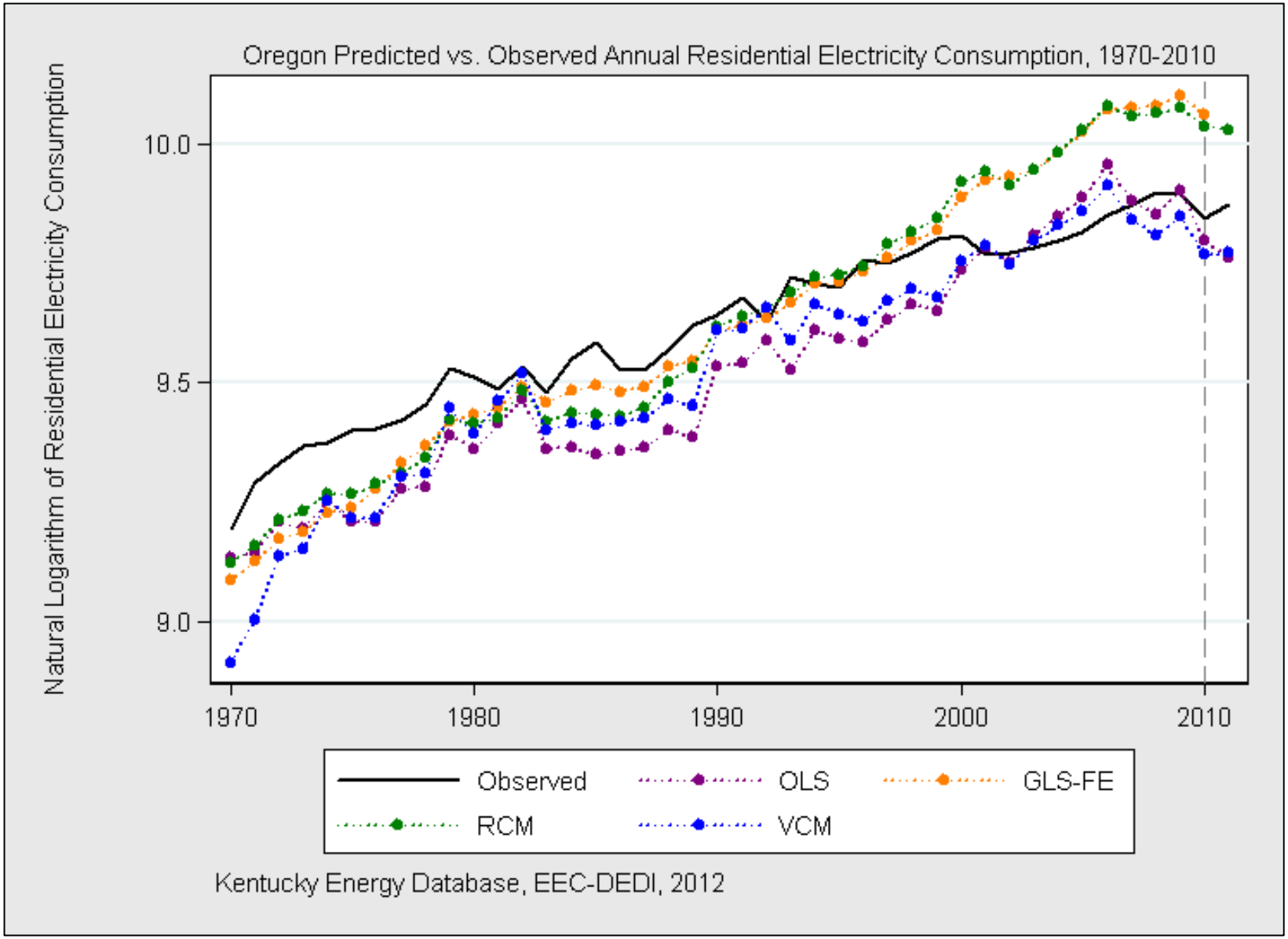
Model Comparison



Model Comparison



Model Comparison



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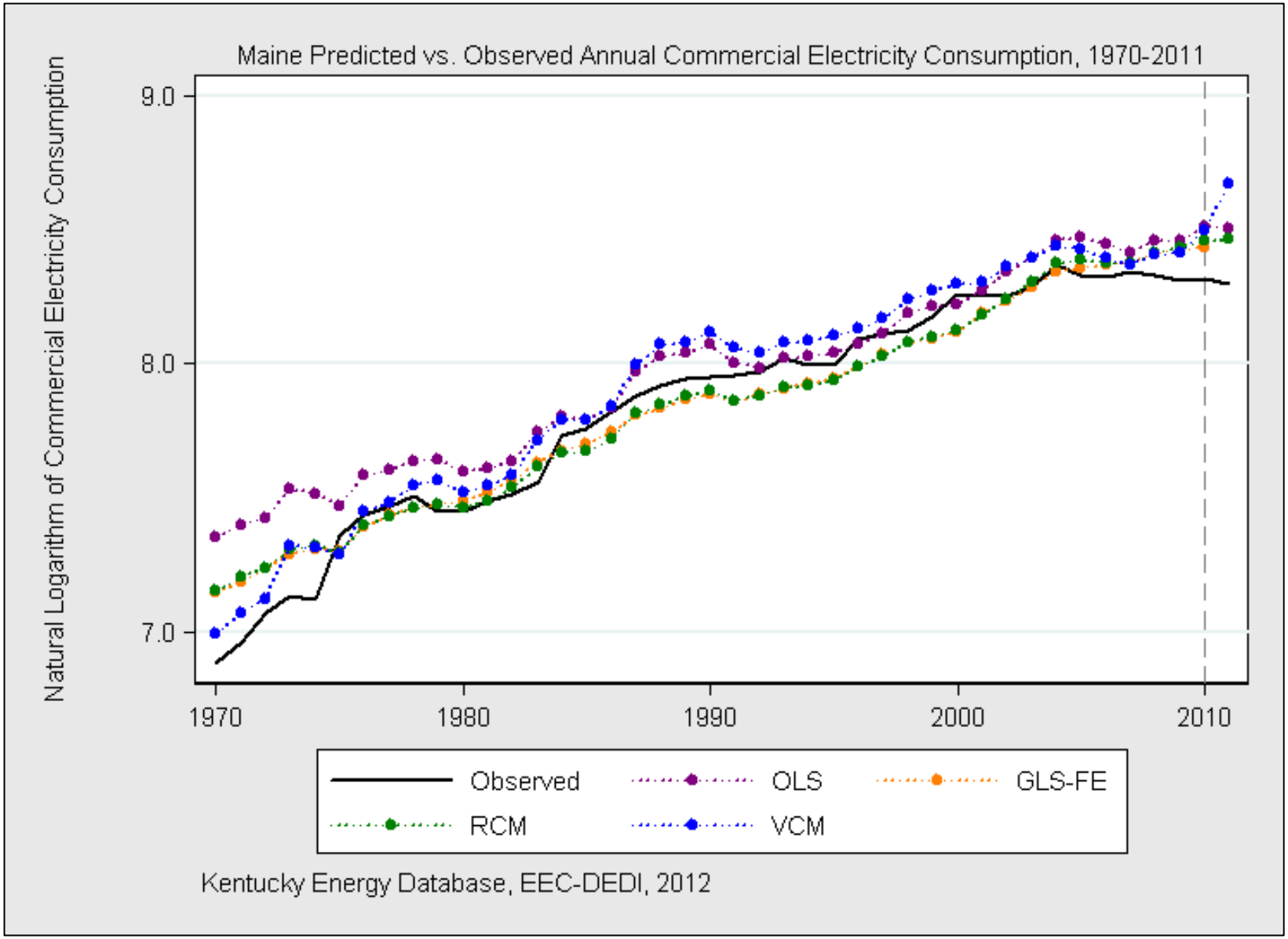

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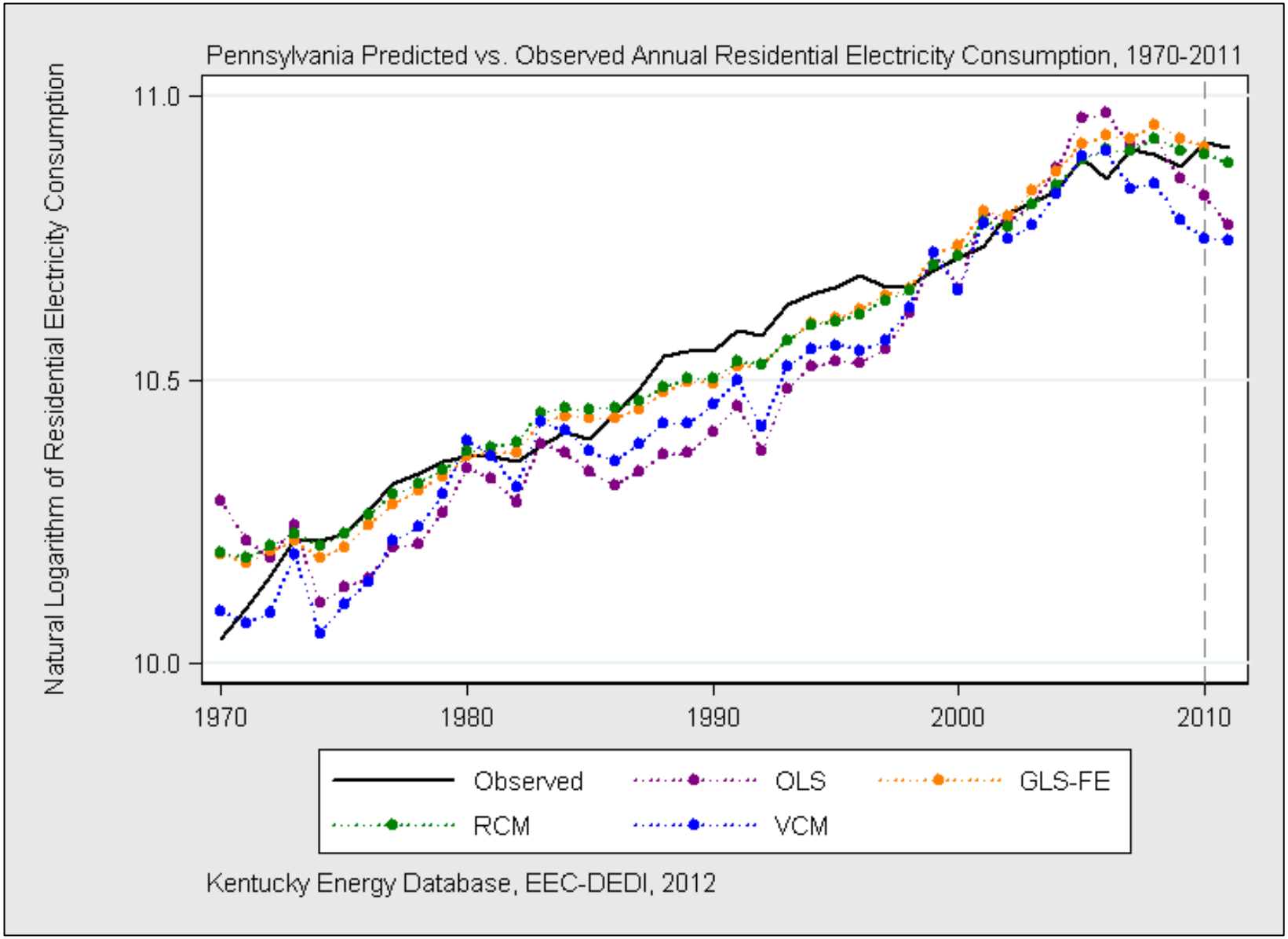

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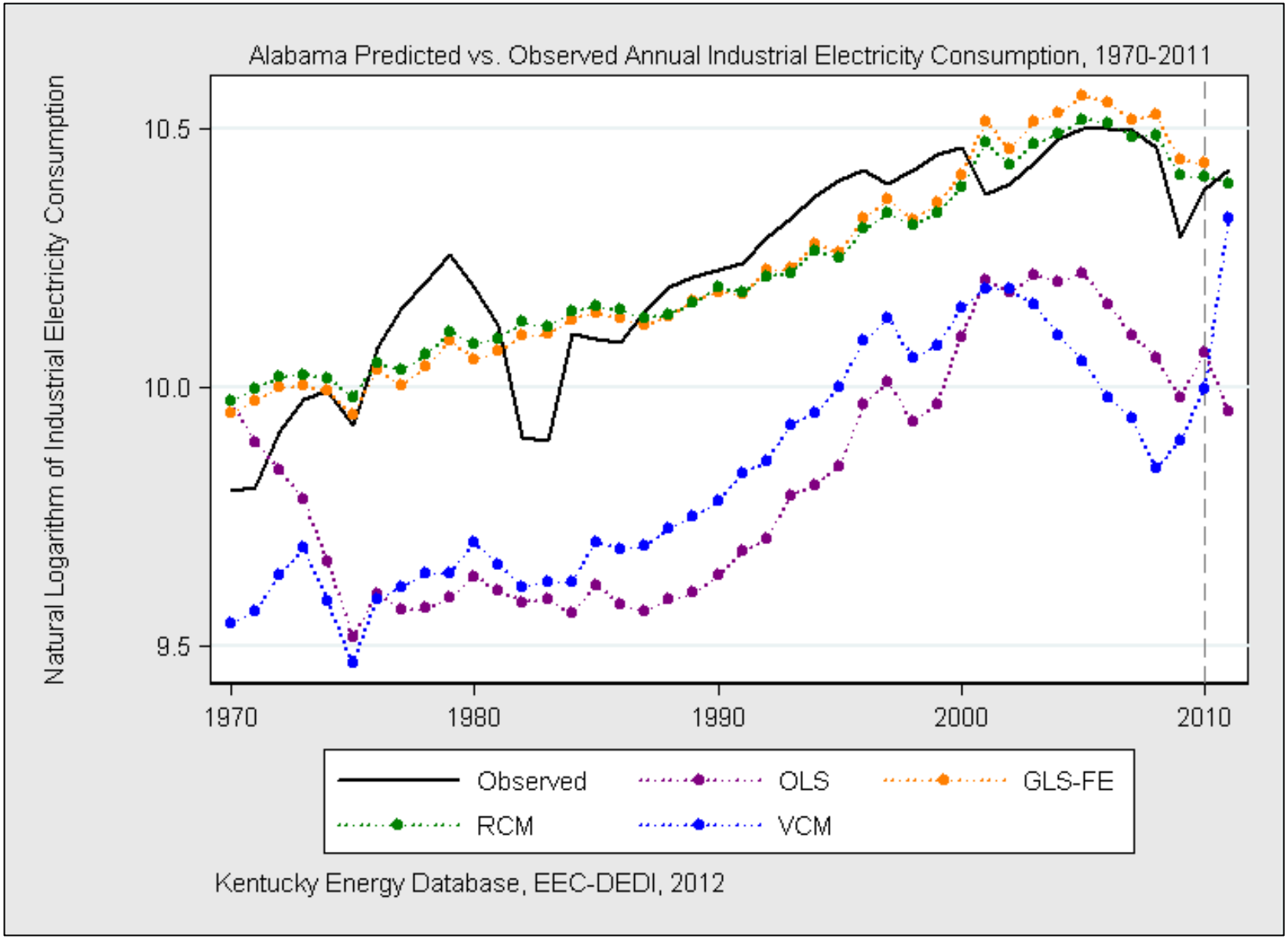

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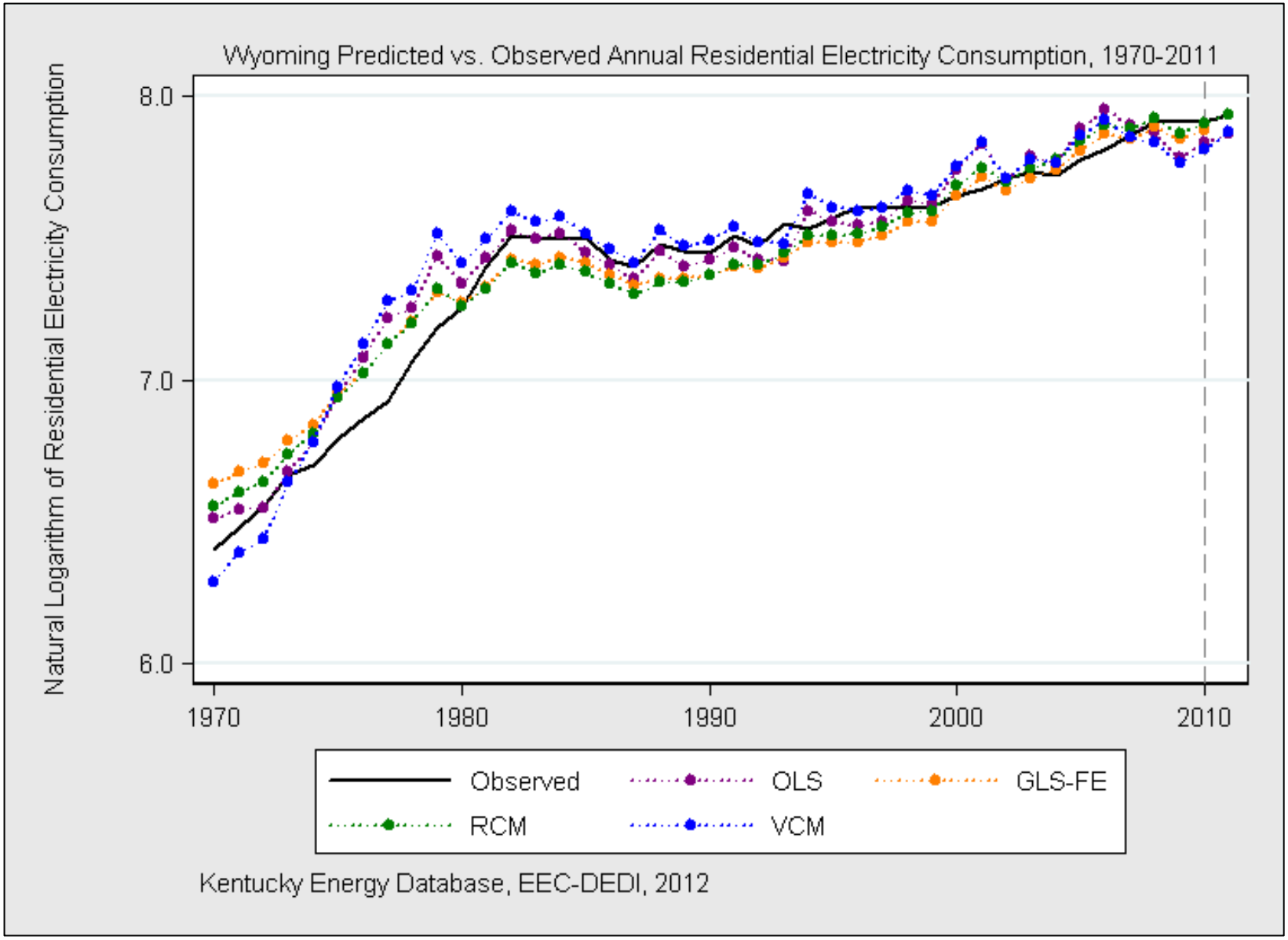

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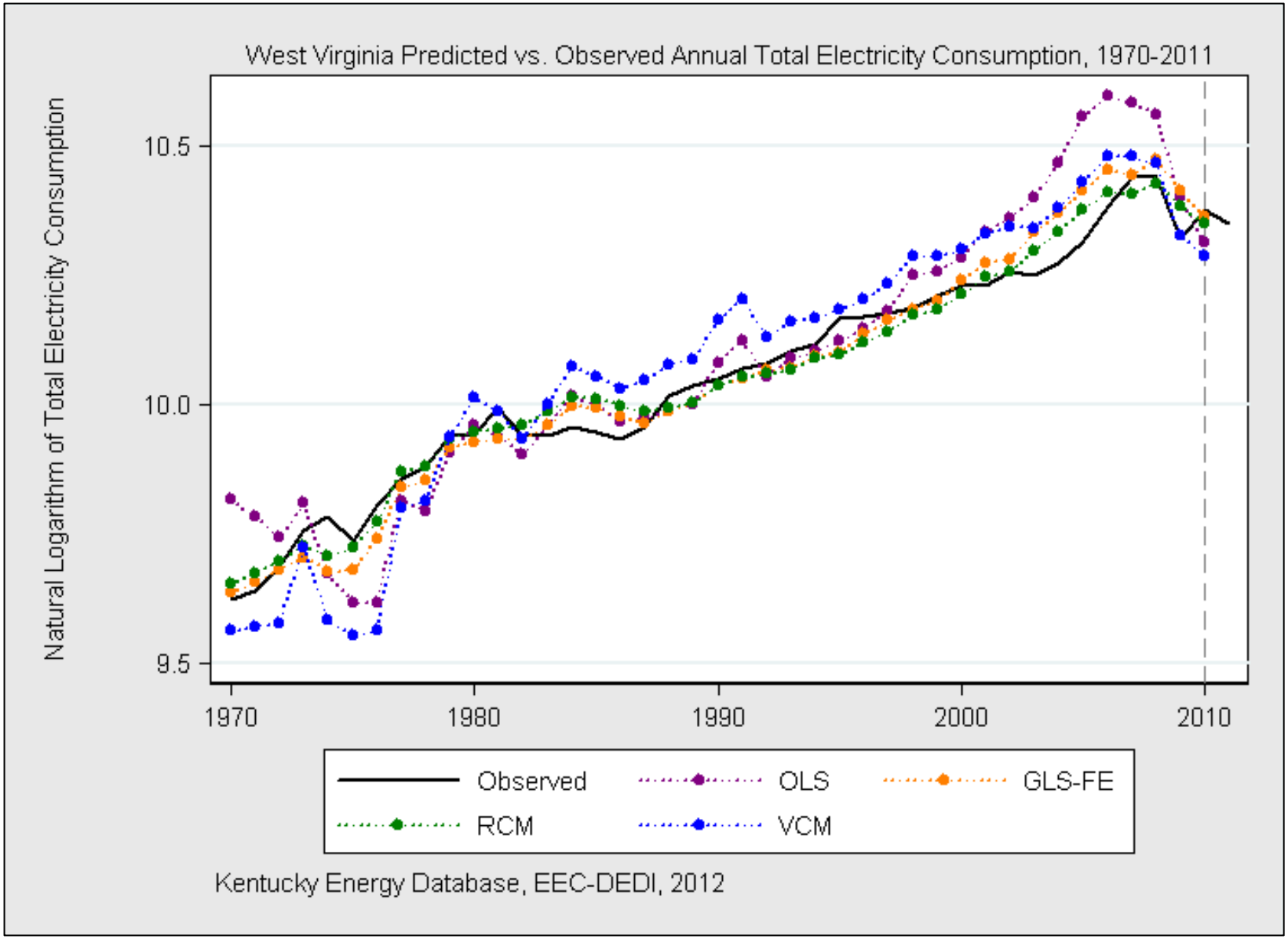

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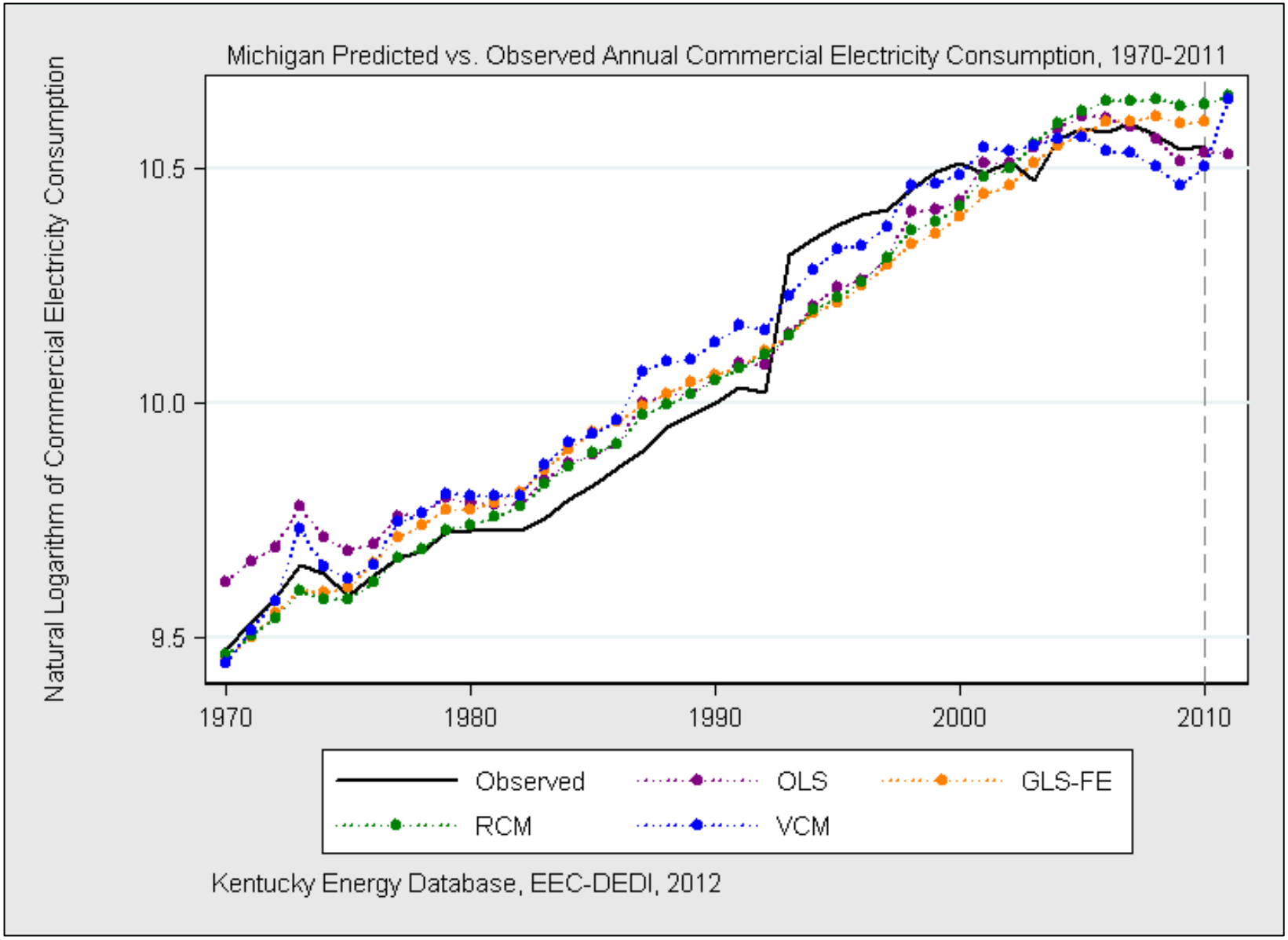

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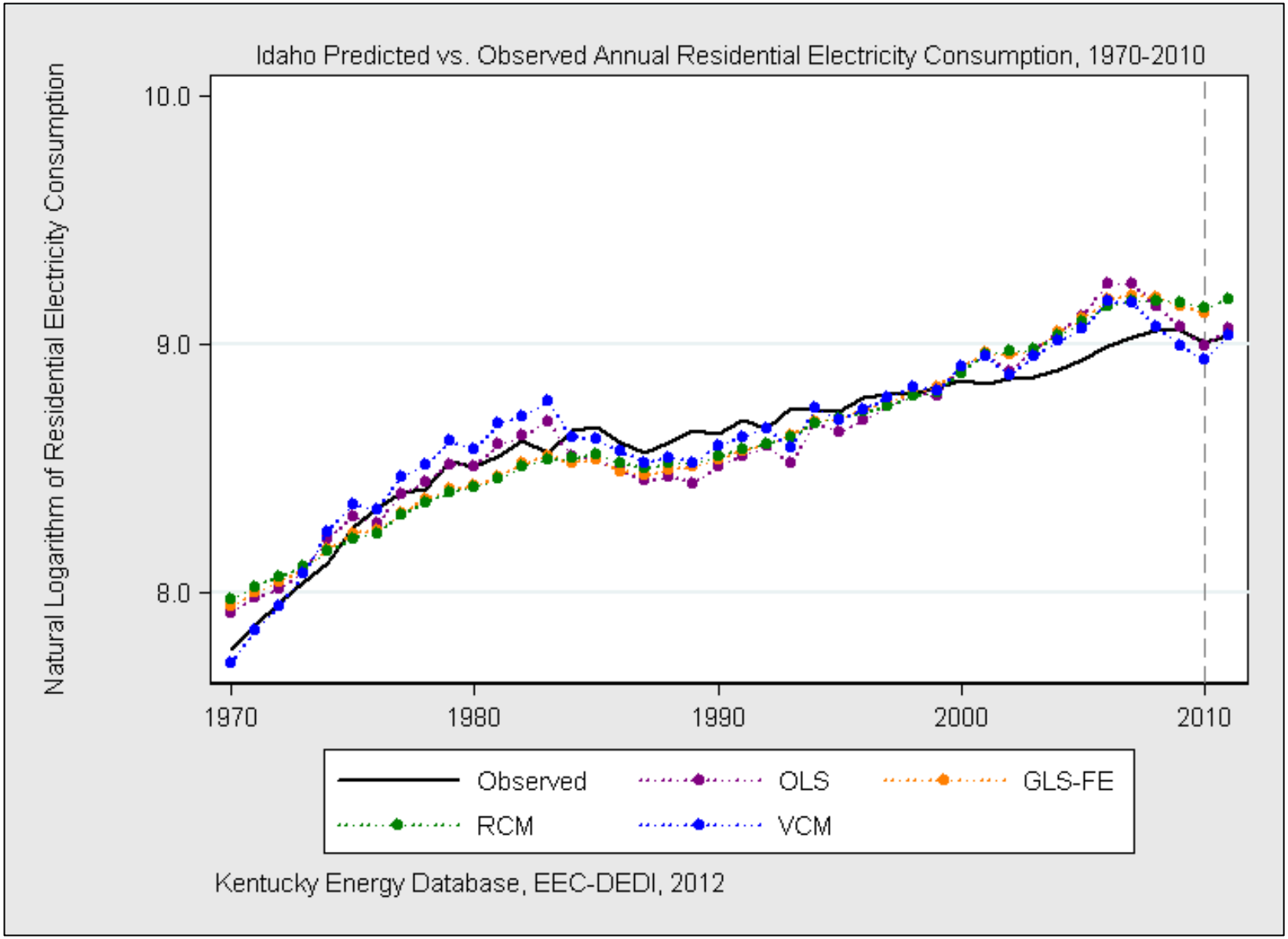

An interactive spreadsheet version of this model is available for download at:

http://energy.ky.gov/Programs/Data%20Analysis%20%20Electricity%20Model/Residential_Electricity_Demand_Model.zip

Questions?

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